

Mathematica 11.3 Integration Test Results

Test results for the 220 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x] (a + b \text{Sech}[c + d x]^2) dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$-\frac{(a + b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{d} + \frac{b \text{Sech}[c + d x]}{d}$$

Result (type 3, 84 leaves):

$$-\frac{a \text{Log}[\text{Cosh}[\frac{c}{2} + \frac{dx}{2}]]}{d} - \frac{b \text{Log}[\text{Cosh}[\frac{1}{2}(c + d x)]]}{d} +$$

$$\frac{a \text{Log}[\text{Sinh}[\frac{c}{2} + \frac{dx}{2}]]}{d} + \frac{b \text{Log}[\text{Sinh}[\frac{1}{2}(c + d x)]]}{d} + \frac{b \text{Sech}[c + d x]}{d}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^3 (a + b \text{Sech}[c + d x]^2) dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{(a + 3 b) \text{ArcTanh}[\text{Cosh}[c + d x]]}{2 d} - \frac{(a + b) \text{Coth}[c + d x] \text{Csch}[c + d x]}{2 d} - \frac{b \text{Sech}[c + d x]}{d}$$

Result (type 3, 169 leaves):

$$-\frac{a \text{Csch}[\frac{1}{2}(c + d x)]^2}{8 d} - \frac{b \text{Csch}[\frac{1}{2}(c + d x)]^2}{8 d} + \frac{a \text{Log}[\text{Cosh}[\frac{1}{2}(c + d x)]]}{2 d} +$$

$$\frac{3 b \text{Log}[\text{Cosh}[\frac{1}{2}(c + d x)]]}{2 d} - \frac{a \text{Log}[\text{Sinh}[\frac{1}{2}(c + d x)]]}{2 d} - \frac{3 b \text{Log}[\text{Sinh}[\frac{1}{2}(c + d x)]]}{2 d} -$$

$$\frac{a \text{Sech}[\frac{1}{2}(c + d x)]^2}{8 d} - \frac{b \text{Sech}[\frac{1}{2}(c + d x)]^2}{8 d} - \frac{b \text{Sech}[c + d x]}{d}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x] (a + b \text{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$-\frac{(a + b)^2 \text{ArcTanh}[\text{Cosh}[c + d x]]}{d} + \frac{b(2a + b) \text{Sech}[c + d x]}{d} + \frac{b^2 \text{Sech}[c + d x]^3}{3d}$$

Result (type 3, 108 leaves):

$$-\left(\left(4(b + a \text{Cosh}[c + d x]^2)^2 \left(-b^2 - 3b(2a + b) \text{Cosh}[c + d x]^2 + 3(a + b)^2 \text{Cosh}[c + d x]^3 \left(\text{Log}[\text{Cosh}[\frac{1}{2}(c + d x)]] - \text{Log}[\text{Sinh}[\frac{1}{2}(c + d x)]] \right) \right) \right) \text{Sech}[c + d x]^3 \right) / \left(3d(a + 2b + a \text{Cosh}[2(c + d x)])^2 \right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^2 (a + b \text{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$-\frac{(a + b)^2 \text{Coth}[c + d x]}{d} - \frac{2b(a + b) \text{Tanh}[c + d x]}{d} + \frac{b^2 \text{Tanh}[c + d x]^3}{3d}$$

Result (type 3, 109 leaves):

$$-\left(\left(4(b + a \text{Cosh}[c + d x]^2)^2 \text{Sech}[c + d x]^3 \left(b^2 \text{Sech}[c] \text{Sinh}[d x] + \text{Cosh}[c + d x]^2 \left(-3(a + b)^2 \text{Coth}[c + d x] \text{Csch}[c] + b(6a + 5b) \text{Sech}[c] \right) \text{Sinh}[d x] + b^2 \text{Cosh}[c + d x] \text{Tanh}[c] \right) \right) \right) / \left(3d(a + 2b + a \text{Cosh}[2(c + d x)])^2 \right)$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \text{Csch}[c + d x]^4 (a + b \text{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$\frac{(a + b)(a + 3b) \text{Coth}[c + d x]}{d} - \frac{(a + b)^2 \text{Coth}[c + d x]^3}{3d} + \frac{b(2a + 3b) \text{Tanh}[c + d x]}{d} - \frac{b^2 \text{Tanh}[c + d x]^3}{3d}$$

Result (type 3, 151 leaves):

$$\begin{aligned}
 & -\frac{1}{6d} \text{Csch}[2c] \text{Csch}[2(c+dx)]^3 \\
 & \left(8a(a+2b) \text{Sinh}[2c] - 6(a+2b)^2 \text{Sinh}[2dx] - 3a^2 \text{Sinh}[2(c+dx)] - \right. \\
 & \quad 6ab \text{Sinh}[2(c+dx)] + a^2 \text{Sinh}[6(c+dx)] + 2ab \text{Sinh}[6(c+dx)] + \\
 & \quad \left. 3a^2 \text{Sinh}[4c+2dx] + a^2 \text{Sinh}[4c+6dx] + 8ab \text{Sinh}[4c+6dx] + 8b^2 \text{Sinh}[4c+6dx] \right)
 \end{aligned}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int (a + b \text{Sech}[c + dx]^2)^3 \text{Sinh}[c + dx]^4 dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$\begin{aligned}
 & \frac{3}{8} a (a^2 - 12ab + 8b^2) x - \frac{3a(a^2 - 12ab + 8b^2) \text{Tanh}[c + dx]}{8d} + \frac{b(6a^2 - 23ab - 8b^2) \text{Tanh}[c + dx]^3}{8d} \\
 & \frac{3(5a - 16b)b^2 \text{Tanh}[c + dx]^5}{40d} - \frac{3(a - 2b) \text{Sinh}[c + dx]^2 \text{Tanh}[c + dx] (a + b - b \text{Tanh}[c + dx]^2)^2}{8d} + \\
 & \frac{\text{Cosh}[c + dx] \text{Sinh}[c + dx]^3 (a + b - b \text{Tanh}[c + dx]^2)^3}{4d}
 \end{aligned}$$

Result (type 3, 651 leaves):

$$\begin{aligned}
 & \frac{1}{1280d(a+2b+a \text{Cosh}[2(c+dx)])^3} (b+a \text{Cosh}[c+dx]^2)^3 \text{Sech}[c] \text{Sech}[c+dx]^5 \\
 & (1200a(a^2-12ab+8b^2)dx \text{Cosh}[dx] + 1200a(a^2-12ab+8b^2)dx \text{Cosh}[2c+dx] + \\
 & \quad 600a^3dx \text{Cosh}[2c+3dx] - 7200a^2bdx \text{Cosh}[2c+3dx] + 4800a^2b^2dx \text{Cosh}[2c+3dx] + \\
 & \quad 600a^3dx \text{Cosh}[4c+3dx] - 7200a^2bdx \text{Cosh}[4c+3dx] + 4800a^2b^2dx \text{Cosh}[4c+3dx] + \\
 & \quad 120a^3dx \text{Cosh}[4c+5dx] - 1440a^2bdx \text{Cosh}[4c+5dx] + 960a^2b^2dx \text{Cosh}[4c+5dx] + \\
 & \quad 120a^3dx \text{Cosh}[6c+5dx] - 1440a^2bdx \text{Cosh}[6c+5dx] + 960a^2b^2dx \text{Cosh}[6c+5dx] - \\
 & \quad 180a^3 \text{Sinh}[dx] + 12120a^2b \text{Sinh}[dx] - 14080a^2b^2 \text{Sinh}[dx] + 1280b^3 \text{Sinh}[dx] - \\
 & \quad 180a^3 \text{Sinh}[2c+dx] - 7080a^2b \text{Sinh}[2c+dx] + 11520a^2b^2 \text{Sinh}[2c+dx] - \\
 & \quad 310a^3 \text{Sinh}[2c+3dx] + 8760a^2b \text{Sinh}[2c+3dx] - 8960a^2b^2 \text{Sinh}[2c+3dx] - \\
 & \quad 310a^3 \text{Sinh}[4c+3dx] - 840a^2b \text{Sinh}[4c+3dx] + 3840a^2b^2 \text{Sinh}[4c+3dx] - \\
 & \quad 640b^3 \text{Sinh}[4c+3dx] - 150a^3 \text{Sinh}[4c+5dx] + 2520a^2b \text{Sinh}[4c+5dx] - \\
 & \quad 2560a^2b^2 \text{Sinh}[4c+5dx] + 128b^3 \text{Sinh}[4c+5dx] - 150a^3 \text{Sinh}[6c+5dx] + \\
 & \quad 600a^2b \text{Sinh}[6c+5dx] - 15a^3 \text{Sinh}[6c+7dx] + 120a^2b \text{Sinh}[6c+7dx] - \\
 & \quad 15a^3 \text{Sinh}[8c+7dx] + 120a^2b \text{Sinh}[8c+7dx] + 5a^3 \text{Sinh}[8c+9dx] + 5a^3 \text{Sinh}[10c+9dx])
 \end{aligned}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int (a + b \text{Sech}[c + dx]^2)^3 \text{Sinh}[c + dx]^2 dx$$

Optimal (type 3, 112 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{1}{2} a^2 (a - 6b) x + \frac{a^3}{4d(1 - \text{Tanh}[c + dx])} - \frac{3a^2 b \text{Tanh}[c + dx]}{d} + \\
 & \frac{b^2(3a + b) \text{Tanh}[c + dx]^3}{3d} - \frac{b^3 \text{Tanh}[c + dx]^5}{5d} - \frac{a^3}{4d(1 + \text{Tanh}[c + dx])}
 \end{aligned}$$

Result (type 3, 480 leaves):

$$\frac{1}{3840 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^5$$

$$\begin{aligned} & (-600 a^2 (a - 6 b) d x \operatorname{Cosh}[d x] - 600 a^2 (a - 6 b) d x \operatorname{Cosh}[2 c + d x] - 300 a^3 d x \operatorname{Cosh}[2 c + 3 d x] + \\ & 1800 a^2 b d x \operatorname{Cosh}[2 c + 3 d x] - 300 a^3 d x \operatorname{Cosh}[4 c + 3 d x] + 1800 a^2 b d x \operatorname{Cosh}[4 c + 3 d x] - \\ & 60 a^3 d x \operatorname{Cosh}[4 c + 5 d x] + 360 a^2 b d x \operatorname{Cosh}[4 c + 5 d x] - 60 a^3 d x \operatorname{Cosh}[6 c + 5 d x] + \\ & 360 a^2 b d x \operatorname{Cosh}[6 c + 5 d x] + 75 a^3 \operatorname{Sinh}[d x] - 4320 a^2 b \operatorname{Sinh}[d x] + 960 a b^2 \operatorname{Sinh}[d x] - \\ & 160 b^3 \operatorname{Sinh}[d x] + 75 a^3 \operatorname{Sinh}[2 c + d x] + 2880 a^2 b \operatorname{Sinh}[2 c + d x] - 1440 a b^2 \operatorname{Sinh}[2 c + d x] - \\ & 480 b^3 \operatorname{Sinh}[2 c + d x] + 135 a^3 \operatorname{Sinh}[2 c + 3 d x] - 2880 a^2 b \operatorname{Sinh}[2 c + 3 d x] + \\ & 480 a b^2 \operatorname{Sinh}[2 c + 3 d x] + 160 b^3 \operatorname{Sinh}[2 c + 3 d x] + 135 a^3 \operatorname{Sinh}[4 c + 3 d x] + \\ & 720 a^2 b \operatorname{Sinh}[4 c + 3 d x] - 720 a b^2 \operatorname{Sinh}[4 c + 3 d x] + 75 a^3 \operatorname{Sinh}[4 c + 5 d x] - \\ & 720 a^2 b \operatorname{Sinh}[4 c + 5 d x] + 240 a b^2 \operatorname{Sinh}[4 c + 5 d x] + 32 b^3 \operatorname{Sinh}[4 c + 5 d x] + \\ & 75 a^3 \operatorname{Sinh}[6 c + 5 d x] + 15 a^3 \operatorname{Sinh}[6 c + 7 d x] + 15 a^3 \operatorname{Sinh}[8 c + 7 d x]) \end{aligned}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{(a + b)^3 \operatorname{Coth}[c + d x]}{d} - \frac{3 b (a + b)^2 \operatorname{Tanh}[c + d x]}{d} + \frac{b^2 (a + b) \operatorname{Tanh}[c + d x]^3}{d} - \frac{b^3 \operatorname{Tanh}[c + d x]^5}{5 d}$$

Result (type 3, 380 leaves):

$$-\frac{1}{40 d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3}$$

$$\begin{aligned} & \operatorname{Coth}[c + d x] \operatorname{Csch}[c] \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 (10 a (5 a^2 + 12 a b + 8 b^2) \operatorname{Sinh}[2 c] - \\ & 10 (5 a^3 + 18 a^2 b + 20 a b^2 + 8 b^3) \operatorname{Sinh}[2 d x] - 25 a^3 \operatorname{Sinh}[2 (c + d x)] + \\ & 50 a b^2 \operatorname{Sinh}[2 (c + d x)] + 30 b^3 \operatorname{Sinh}[2 (c + d x)] - 20 a^3 \operatorname{Sinh}[4 (c + d x)] + \\ & 40 a b^2 \operatorname{Sinh}[4 (c + d x)] + 24 b^3 \operatorname{Sinh}[4 (c + d x)] - 5 a^3 \operatorname{Sinh}[6 (c + d x)] + \\ & 10 a b^2 \operatorname{Sinh}[6 (c + d x)] + 6 b^3 \operatorname{Sinh}[6 (c + d x)] - 25 a^3 \operatorname{Sinh}[2 (c + 2 d x)] - \\ & 120 a^2 b \operatorname{Sinh}[2 (c + 2 d x)] - 160 a b^2 \operatorname{Sinh}[2 (c + 2 d x)] - 64 b^3 \operatorname{Sinh}[2 (c + 2 d x)] + \\ & 25 a^3 \operatorname{Sinh}[4 c + 2 d x] + 30 a^2 b \operatorname{Sinh}[4 c + 2 d x] + 5 a^3 \operatorname{Sinh}[6 c + 4 d x] - 5 a^3 \operatorname{Sinh}[4 c + 6 d x] - \\ & 30 a^2 b \operatorname{Sinh}[4 c + 6 d x] - 40 a b^2 \operatorname{Sinh}[4 c + 6 d x] - 16 b^3 \operatorname{Sinh}[4 c + 6 d x]) \end{aligned}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c + d x]^3 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{(a+b)^2 (a+7b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2d} - \frac{(a+b)^2 (a+7b) \operatorname{Sech}[c+dx]}{2d} -$$

$$\frac{b(6a^2+15ab+7b^2) \operatorname{Sech}[c+dx]^3}{6d} - \frac{b^2(5a+7b) \operatorname{Sech}[c+dx]^5}{10d} -$$

$$\frac{(a+b)(b+a \operatorname{Cosh}[c+dx])^2 \operatorname{Csch}[c+dx]^2 \operatorname{Sech}[c+dx]^5}{2d}$$

Result (type 3, 409 leaves):

$$-\frac{1}{120d(a+2b+a \operatorname{Cosh}[2c+2dx])^3}$$

$$(150a^3+270a^2b-30ab^2-206b^3+225a^3 \operatorname{Cosh}[2c+2dx]+585a^2b \operatorname{Cosh}[2c+2dx]+$$

$$495ab^2 \operatorname{Cosh}[2c+2dx]+231b^3 \operatorname{Cosh}[2c+2dx]+90a^3 \operatorname{Cosh}[4c+4dx]+$$

$$450a^2b \operatorname{Cosh}[4c+4dx]+750ab^2 \operatorname{Cosh}[4c+4dx]+350b^3 \operatorname{Cosh}[4c+4dx]+$$

$$15a^3 \operatorname{Cosh}[6c+6dx]+135a^2b \operatorname{Cosh}[6c+6dx]+225ab^2 \operatorname{Cosh}[6c+6dx]+$$

$$105b^3 \operatorname{Cosh}[6c+6dx]) \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx] (a+b \operatorname{Sech}[c+dx]^2)^3 +$$

$$\left(4(a^3+9a^2b+15ab^2+7b^3) \operatorname{Cosh}[c+dx]^6 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (a+b \operatorname{Sech}[c+dx]^2)^3\right) /$$

$$(d(a+2b+a \operatorname{Cosh}[2c+2dx])^3) -$$

$$\left(4(a^3+9a^2b+15ab^2+7b^3) \operatorname{Cosh}[c+dx]^6 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{c}{2}+\frac{dx}{2}\right]\right] (a+b \operatorname{Sech}[c+dx]^2)^3\right) /$$

$$(d(a+2b+a \operatorname{Cosh}[2c+2dx])^3)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csch}[c+dx]^4 (a+b \operatorname{Sech}[c+dx]^2)^3 dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{(a+b)^2 (a+4b) \operatorname{Coth}[c+dx]}{d} - \frac{(a+b)^3 \operatorname{Coth}[c+dx]^3}{3d} +$$

$$\frac{3b(a+b)(a+2b) \operatorname{Tanh}[c+dx]}{d} - \frac{b^2(3a+4b) \operatorname{Tanh}[c+dx]^3}{3d} + \frac{b^3 \operatorname{Tanh}[c+dx]^5}{5d}$$

Result (type 3, 213 leaves):

$$-\frac{1}{15d(a+2b+a \operatorname{Cosh}[2(c+dx)])^3} 8(b+a \operatorname{Cosh}[c+dx]^2)^3 \operatorname{Sech}[c+dx]^5$$

$$\left(-3b^3 \operatorname{Cosh}[c+dx] + \operatorname{Cosh}[c+dx]^3 \left(-b^2(15a+14b) + 5(a+b)^3 \operatorname{Coth}[c]^2 \operatorname{Coth}[c+dx]^2\right) -$$

$$3b^3 \operatorname{Csch}[c] \operatorname{Sinh}[dx] + \operatorname{Cosh}[c+dx]^4 \left(-b(45a^2+120ab+73b^2) +$$

$$5(a+b)^2(2a+11b) \operatorname{Coth}[c] \operatorname{Coth}[c+dx]\right) \operatorname{Csch}[c] \operatorname{Sinh}[dx] -$$

$$\operatorname{Cosh}[c+dx]^2 \left(b^2(15a+14b) + 5(a+b)^3 \operatorname{Coth}[c] \operatorname{Coth}[c+dx]^3\right) \operatorname{Csch}[c] \operatorname{Sinh}[dx]\right) \operatorname{Tanh}[c]$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]^4}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{(3 a^2 + 12 a b + 8 b^2) x}{8 a^3} - \frac{\sqrt{b} (a + b)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a^3 d} - \frac{(5 a + 4 b) \text{Cosh}[c + d x] \text{Sinh}[c + d x]}{8 a^2 d} + \frac{\text{Cosh}[c + d x]^3 \text{Sinh}[c + d x]}{4 a d}$$

Result (type 3, 294 leaves):

$$\frac{1}{64 a^3 \sqrt{b} \sqrt{a + b} d (a + b \text{Sech}[c + d x]^2) \sqrt{b} (\text{Cosh}[c] - \text{Sinh}[c])^4} \left((a + 2 b + a \text{Cosh}[2(c + d x)]) \text{Sech}[c + d x]^2 \left(\sqrt{b} (3 a^3 + 34 a^2 b + 64 a b^2 + 32 b^3) \text{ArcTanh}\left[\frac{\text{Sech}[d x] (\text{Cosh}[2 c] - \text{Sinh}[2 c]) ((a + 2 b) \text{Sinh}[d x] - a \text{Sinh}[2 c + d x])}{2 \sqrt{a + b} \sqrt{b} (\text{Cosh}[c] - \text{Sinh}[c])^4} \right] (\text{Cosh}[2 c] - \text{Sinh}[2 c]) - \sqrt{b} (\text{Cosh}[c] - \text{Sinh}[c])^4 \left(a^2 (3 a + 2 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}} \right] + \sqrt{b} \sqrt{a + b} (-2 a^2 c + 12 a^2 d x + 48 a b d x + 32 b^2 d x - 8 a (a + b) \text{Sinh}[2(c + d x)] + a^2 \text{Sinh}[4(c + d x)]) \right) \right) \right)$$

Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]^3}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\sqrt{b} (a + b) \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c + d x]}{\sqrt{b}}\right]}{a^{5/2} d} - \frac{(a + b) \text{Cosh}[c + d x]}{a^2 d} + \frac{\text{Cosh}[c + d x]^3}{3 a d}$$

Result (type 3, 372 leaves):

$$\frac{1}{48 a^{5/2} \sqrt{b} d (b + a \operatorname{Cosh}[c + d x]^2)} (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])$$

$$\left(3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right. \right. \right.$$

$$\left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$3 (a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) -$$

$$3 a^2 \left(\operatorname{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] \right) -$$

$$6 \sqrt{a} \sqrt{b} (3 a + 4 b) \operatorname{Cosh}[c + d x] + 2 a^{3/2} \sqrt{b} \operatorname{Cosh}[3 (c + d x)] \Bigg)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + d x]^2}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 75 leaves, 5 steps):

$$-\frac{(a + 2 b) x}{2 a^2} + \frac{\sqrt{b} \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{a^2 d} + \frac{\operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d}$$

Result (type 3, 236 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^2 \right.$$

$$\left(-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{\sqrt{b} \sqrt{a+b} d} + \frac{1}{a^2} \left(-4 (a + 2 b) x + \left((a^2 + 8 a b + 8 b^2) \right. \right. \right.$$

$$\left. \left. \operatorname{ArcTanh}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])) \right] \right) \right) \Bigg) /$$

$$\left(2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) \Bigg) /$$

$$\left(\sqrt{a+b} d \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \frac{2 a \operatorname{Cosh}[2 d x] \operatorname{Sinh}[2 c]}{d} +$$

$$\left. \left. \frac{2 a \operatorname{Cosh}[2 c] \operatorname{Sinh}[2 d x]}{d} \right) \Bigg) / (16 (a + b \operatorname{Sech}[c + d x]^2))$$

Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$-\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c+dx]}{\sqrt{b}}\right]}{a^{3/2} d} + \frac{\text{Cosh}[c + d x]}{a d}$$

Result (type 3, 328 leaves):

$$\frac{1}{8 a^{3/2} d (a + b \text{Sech}[c + d x]^2)} \left(-\frac{1}{\sqrt{b}} (a + 4 b) \left(\text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) + \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) + \frac{1}{\sqrt{b}} a \left(\text{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] \right) + 4 \sqrt{a} \text{Cosh}[c + d x] \right) \text{Sech}[c + d x]^2$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c+dx]}{\sqrt{b}}\right]}{\sqrt{a} (a + b) d} - \frac{\text{ArcTanh}[\text{Cosh}[c + d x]]}{(a + b) d}$$

Result (type 3, 232 leaves):

$$\frac{1}{(a+b)d} \left(\frac{1}{\sqrt{a}} \sqrt{b} \operatorname{ArcTan} \left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh} \left[\frac{dx}{2} \right] + \operatorname{Cosh}[c] \right. \right. \right. \right. \\ \left. \left. \left. \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh} \left[\frac{dx}{2} \right] \right) \right] \right) + \frac{1}{\sqrt{a}} \right. \\ \left. \sqrt{b} \operatorname{ArcTan} \left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh} \left[\frac{dx}{2} \right] + \right. \right. \right. \right. \\ \left. \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh} \left[\frac{dx}{2} \right] \right) \right) \right] \right) - \\ \left. \operatorname{Log} \left[\operatorname{Cosh} \left[\frac{1}{2} (c+dx) \right] \right] + \operatorname{Log} \left[\operatorname{Sinh} \left[\frac{1}{2} (c+dx) \right] \right] \right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^2}{a+b \operatorname{Sech}[c+dx]^2} dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2} d} - \frac{\operatorname{Coth}[c+dx]}{(a+b)d}$$

Result (type 3, 179 leaves):

$$\left((a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^2 \right. \\ \left(b \operatorname{ArcTanh} \left[\left(\operatorname{Sech}[dx] \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \left((a+2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c+dx] \right) \right) \right] \right) / \\ \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) + \\ \left. \sqrt{a+b} \operatorname{Csch}[c] \operatorname{Csch}[c+dx] \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \operatorname{Sinh}[dx] \right) \right) / \\ \left(2 (a+b)^{3/2} d (a+b \operatorname{Sech}[c+dx]^2) \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right)$$

Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^3}{a+b \operatorname{Sech}[c+dx]^2} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$-\frac{\sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Cosh}[c+dx]}{\sqrt{b}} \right]}{(a+b)^2 d} + \frac{(a-b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{2(a+b)^2 d} - \frac{\operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]}{2(a+b)d}$$

Result (type 3, 338 leaves):

$$\begin{aligned}
 & - \frac{1}{16 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)} (a+2 b+a \operatorname{Cosh}[2(c+d x)]) \\
 & \left(8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(\sqrt{a}-i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right]+ \right.\right.\right. \\
 & \quad \left.\left.\left.\operatorname{Cosh}[c]\left(\sqrt{a}-i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right]\right)\right]\right)\right] + \\
 & 8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(\sqrt{a}+i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}\right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right]+ \right.\right. \\
 & \quad \left.\left.\left.\operatorname{Cosh}[c]\left(\sqrt{a}+i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right]\right)\right]\right)\right] + \\
 & (a+b) \operatorname{CsCh}\left[\frac{1}{2}(c+d x)\right]^2-4 a \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]+4 b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right]+ \\
 & 4 a \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]-4 b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right]+ \\
 & (a+b) \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sech}[c+d x]^2
 \end{aligned}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{CsCh}[c+d x]^4}{a+b \operatorname{Sech}[c+d x]^2} d x$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2} d}+\frac{a \operatorname{Coth}[c+d x]}{(a+b)^2 d}-\frac{\operatorname{Coth}[c+d x]^3}{3(a+b) d}$$

Result (type 3, 216 leaves):

$$\begin{aligned}
 & \left((a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^2 \right. \\
 & \left. \left(3 a b \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[d x]\left(\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c]\right)\left((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x]\right)\right]\right] \right. \right. \\
 & \quad \left. \left. \left(2 \sqrt{a+b} \sqrt{b\left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c]\right)^4}\right)\left(-\operatorname{Cosh}[2 c]+\operatorname{Sinh}[2 c]\right)+ \right.\right. \\
 & \quad \left. \left. \frac{1}{4} \sqrt{a+b} \operatorname{CsCh}[c] \operatorname{CsCh}[c+d x]^3 \sqrt{b\left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c]\right)^4} \right.\right. \\
 & \quad \left. \left. \left(6 a \operatorname{Sinh}[d x]-3 b \operatorname{Sinh}[2 c+d x]+(-2 a+b) \operatorname{Sinh}[2 c+3 d x]\right)\right)\right) \Bigg) / \\
 & \left(6(a+b)^{5/2} d(a+b \operatorname{Sech}[c+d x]^2) \sqrt{b\left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c]\right)^4} \right)
 \end{aligned}$$

$$\frac{\frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}}}{\operatorname{Sinh}[2 c+d x]} \operatorname{Cosh}[2 c]}{\left(8 a^4 b \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}\right)} + \left(i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}}\right) (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \operatorname{Sinh}[2 c]\right) / \left(8 a^4 b \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}\right)} + \frac{1}{8 a^4 b (a+b) d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])} \operatorname{Sech}[2 c] \left(160 a^4 b d x \operatorname{Cosh}[2 c]+1248 a^3 b^2 d x \operatorname{Cosh}[2 c]+3392 a^2 b^3 d x \operatorname{Cosh}[2 c]+3840 a b^4 d x \operatorname{Cosh}[2 c]+1536 b^5 d x \operatorname{Cosh}[2 c]+80 a^4 b d x \operatorname{Cosh}[2 d x]+464 a^3 b^2 d x \operatorname{Cosh}[2 d x]+768 a^2 b^3 d x \operatorname{Cosh}[2 d x]+384 a b^4 d x \operatorname{Cosh}[2 d x]+80 a^4 b d x \operatorname{Cosh}[4 c+2 d x]+464 a^3 b^2 d x \operatorname{Cosh}[4 c+2 d x]+768 a^2 b^3 d x \operatorname{Cosh}[4 c+2 d x]+384 a b^4 d x \operatorname{Cosh}[4 c+2 d x]+a^5 \operatorname{Sinh}[2 c]+34 a^4 b \operatorname{Sinh}[2 c]+224 a^3 b^2 \operatorname{Sinh}[2 c]+576 a^2 b^3 \operatorname{Sinh}[2 c]+640 a b^4 \operatorname{Sinh}[2 c]+256 b^5 \operatorname{Sinh}[2 c]-a^5 \operatorname{Sinh}[2 d x]-62 a^4 b \operatorname{Sinh}[2 d x]-318 a^3 b^2 \operatorname{Sinh}[2 d x]-512 a^2 b^3 \operatorname{Sinh}[2 d x]-256 a b^4 \operatorname{Sinh}[2 d x]-30 a^4 b \operatorname{Sinh}[4 c+2 d x]-158 a^3 b^2 \operatorname{Sinh}[4 c+2 d x]-256 a^2 b^3 \operatorname{Sinh}[4 c+2 d x]-128 a b^4 \operatorname{Sinh}[4 c+2 d x]-12 a^4 b \operatorname{Sinh}[2 c+4 d x]-36 a^3 b^2 \operatorname{Sinh}[2 c+4 d x]-24 a^2 b^3 \operatorname{Sinh}[2 c+4 d x]-12 a^4 b \operatorname{Sinh}[6 c+4 d x]-36 a^3 b^2 \operatorname{Sinh}[6 c+4 d x]-24 a^2 b^3 \operatorname{Sinh}[6 c+4 d x]+2 a^4 b \operatorname{Sinh}[4 c+6 d x]+2 a^3 b^2 \operatorname{Sinh}[4 c+6 d x]+2 a^4 b \operatorname{Sinh}[8 c+6 d x]+2 a^3 b^2 \operatorname{Sinh}[8 c+6 d x]\right)$$

Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+d x]^3}{(a+b \operatorname{Sech}[c+d x]^2)^2} dx$$

Optimal (type 3, 114 leaves, 5 steps):

$$\frac{\sqrt{b} (3 a+5 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+d x]}{\sqrt{b}}\right]}{2 a^{7/2} d} - \frac{(a+2 b) \operatorname{Cosh}[c+d x]}{a^3 d} + \frac{\operatorname{Cosh}[c+d x]^3}{3 a^2 d} - \frac{b(a+b) \operatorname{Cosh}[c+d x]}{2 a^3 d (b+a \operatorname{Cosh}[c+d x]^2)}$$

Result (type 3, 861 leaves):

$$\frac{1}{1536 a^{7/2} d (a + b \operatorname{Sech}[c + d x])^2} (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \operatorname{Sech}[c + d x]^4$$

$$\left(\frac{1}{b^{3/2}} 9 a^3 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$576 a \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$960 b^{3/2} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) + \frac{1}{b^{3/2}}$$

$$9 a^3 \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$576 a \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$960 b^{3/2} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) -$$

$$\frac{9 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{9 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right]}{b^{3/2}} -$$

$$96 \sqrt{a} (3 a + 8 b) \operatorname{Cosh}[c] \operatorname{Cosh}[d x] +$$

$$32 a^{3/2} \operatorname{Cosh}[3 c] \operatorname{Cosh}[3 d x] - \frac{384 a^{3/2} b \operatorname{Cosh}[c + d x]}{a + 2 b + a \operatorname{Cosh}[2 (c + d x)]} -$$

$$\frac{384 \sqrt{a} b^2 \operatorname{Cosh}[c + d x]}{a + 2 b + a \operatorname{Cosh}[2 (c + d x)]} - 288 a^{3/2} \operatorname{Sinh}[c] \operatorname{Sinh}[d x] -$$

$$768 \sqrt{a} b \operatorname{Sinh}[c] \operatorname{Sinh}[d x] + 32 a^{3/2} \operatorname{Sinh}[3 c] \operatorname{Sinh}[3 d x]$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]^2}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$-\frac{(a + 4 b) x}{2 a^3} + \frac{\sqrt{b} (3 a + 4 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{2 a^3 \sqrt{a + b} d} + \frac{\text{Cosh}[c + d x] \text{Sinh}[c + d x]}{2 a d (a + b - b \text{Tanh}[c + d x]^2)} + \frac{b \text{Tanh}[c + d x]}{a^2 d (a + b - b \text{Tanh}[c + d x]^2)}$$

Result (type 3, 791 leaves):

$$\begin{aligned}
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left(16x + \left((a^3 - 6a^2b - 24ab^2 - 16b^3) \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx] \right) \right) / \left(2\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) \right] \right) \\
 & \quad \left. \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \right) / \left(b (a+b)^{3/2} d \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \\
 & \quad \left((a^2 + 8ab + 8b^2) \operatorname{Sech}[2c] \left((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx] \right) \right) / \\
 & \quad \left. \left(b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \right) \right) \Bigg) / \\
 & \left(128a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left(-64(a + 2b)x + \left(-a^4 + 16a^3b + 144a^2b^2 + 256ab^3 + 128b^4 \right) \right. \\
 & \quad \left. \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \left((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx] \right) \right] / \right. \\
 & \quad \left. \left(2\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) \right] \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \right) / \\
 & \quad \left(b (a+b)^{3/2} d \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \frac{16a \operatorname{Cosh}[2dx] \operatorname{Sinh}[2c]}{d} + \\
 & \quad \frac{16a \operatorname{Cosh}[2c] \operatorname{Sinh}[2dx]}{d} - \left((a^3 + 18a^2b + 48ab^2 + 32b^3) \operatorname{Sech}[2c] \right. \\
 & \quad \left. \left. \left((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx] \right) \right) / \left(b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \right) \right) \Bigg) / \\
 & \left(256a^3 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) - \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left(- \frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} + \frac{\sqrt{b} (a + 2b) \operatorname{Sinh}[2(c + dx)]}{(a+b) (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \Bigg) / \\
 & \left(256b^{3/2} d (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left(- \frac{(a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}} \right]}{8b^{3/2} (a+b)^{3/2} d} + \frac{a \operatorname{Sinh}[2(c + dx)]}{8b (a+b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \Bigg) / \\
 & \left(16(a + b \operatorname{Sech}[c + dx]^2)^2 \right)
 \end{aligned}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c + d x]}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-\frac{3 \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c+dx]}{\sqrt{b}}\right]}{2 a^{5/2} d} + \frac{3 \text{Cosh}[c + d x]}{2 a^2 d} - \frac{\text{Cosh}[c + d x]^3}{2 a d (b + a \text{Cosh}[c + d x]^2)}$$

Result (type 3, 479 leaves):

$$\frac{1}{128 d (a + b \text{Sech}[c + d x]^2)^2} (a + 2 b + a \text{Cosh}[2 (c + d x)])^2$$

$$\text{Sech}[c + d x]^4 \left(\frac{32 \text{Cosh}[c] \text{Cosh}[d x]}{a^2} + \frac{32 b \text{Cosh}[c + d x]}{a^2 (a + 2 b + a \text{Cosh}[2 (c + d x)])} + \right.$$

$$\left. \frac{1}{a^{5/2} b^{3/2}} 2 \left(- (a^2 + 24 b^2) \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b}) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right) \text{Sinh}[c] \right. \right. \right.$$

$$\left. \left. \text{Tanh}\left[\frac{d x}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) -$$

$$a^2 \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \left. \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) -$$

$$24 b^2 \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b}) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right) \text{Sinh}[c] \text{Tanh}\left[\frac{d x}{2}\right] + \right.$$

$$\left. \left. \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{d x}{2}\right] \right) \right] \right) +$$

$$a^2 \text{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] +$$

$$a^2 \text{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{b}}\right] + 16 \sqrt{a} b^{3/2} \text{Sinh}[c] \text{Sinh}[d x] \Bigg)$$

Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Csch}[c + d x]}{(a + b \text{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$\frac{\sqrt{b} (3a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+dx]}{\sqrt{b}}\right]}{2a^{3/2} (a+b)^2 d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{(a+b)^2 d} - \frac{b \operatorname{Cosh}[c+dx]}{2a(a+b)d(b+a \operatorname{Cosh}[c+dx]^2)}$$

Result (type 3, 377 leaves):

$$\begin{aligned} & \frac{1}{8(a+b)^2 d (a+b \operatorname{Sech}[c+dx]^2)^2} (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^3 \\ & \left(-\frac{2b(a+b)}{a} + \frac{1}{a^{3/2}} \sqrt{b} (3a+b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) \\ & (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] + \frac{1}{a^{3/2}} \sqrt{b} (3a+b) \\ & \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \\ & \quad \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \\ & (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] - 2(a+2b+a \operatorname{Cosh}[2(c+dx)]) \\ & \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] + \\ & 2(a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] \end{aligned}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^2}{(a+b \operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{3\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} d} - \frac{3 \operatorname{Coth}[c+dx]}{2(a+b)^2 d} + \frac{\operatorname{Coth}[c+dx]}{2(a+b)d(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 220 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \right. \\ \left. \left(\left(3b \operatorname{ArcTanh} \left[\left(\operatorname{Sech}[dx] \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \left((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx] \right) \right] \right) \right) \right. \right. \\ \left. \left(2\sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) \left(a + 2b + a \operatorname{Cosh}[2(c + dx)] \right) \right. \\ \left. \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \right) \left(\sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) + \\ 2(a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sinh}[dx] + \\ b \operatorname{Sech}[2c] \operatorname{Sinh}[2dx] - \frac{b(a + 2b) \operatorname{Tanh}[2c]}{a} \right) \left. \right) \left. \right) / \\ \left(8(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right)$$

Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c + dx]^3}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$-\frac{(3a - b) \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Cosh}[c + dx]}{\sqrt{b}} \right]}{2\sqrt{a} (a + b)^3 d} + \frac{(a - 3b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]}{2(a + b)^3 d} - \\ \frac{(a - b) \operatorname{Cosh}[c + dx]}{2(a + b)^2 d (b + a \operatorname{Cosh}[c + dx]^2)} - \frac{\operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]}{2(a + b) d (b + a \operatorname{Cosh}[c + dx]^2)}$$

Result (type 3, 462 leaves):

$$\begin{aligned}
 & \frac{1}{32 (a+b)^3 d (a+b \operatorname{Sech}[c+dx])^2} (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx]^3 \\
 & \left(8b(a+b) + \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a+b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) \\
 & (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] + \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a+b) \\
 & \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \\
 & (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}[c+dx] - (a+b) (a+2b+a \operatorname{Cosh}[2(c+dx)]) \\
 & \operatorname{Csch}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sech}[c+dx] + \\
 & 4(a-3b) (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] - \\
 & 4(a-3b) (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sech}[c+dx] - \\
 & (a+b) (a+2b+a \operatorname{Cosh}[2(c+dx)]) \operatorname{Sech}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sech}[c+dx] \Big)
 \end{aligned}$$

Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx])^2} dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{(3a-2b)\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2(a+b)^{7/2} d} + \\
 & \frac{(a-b) \operatorname{Coth}[c+dx]}{(a+b)^3 d} - \frac{\operatorname{Coth}[c+dx]^3}{3(a+b)^2 d} - \frac{ab \operatorname{Tanh}[c+dx]}{2(a+b)^3 d (a+b-b \operatorname{Tanh}[c+dx])^2}
 \end{aligned}$$

Result (type 3, 620 leaves):

$$\begin{aligned}
 & - \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Coth}[c] \operatorname{Csch}[c + dx]^2 \operatorname{Sech}[c + dx]^4}{12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
 & \left((3a - 2b) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\
 & \quad \left. \left. \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / \\
 & \quad \left(8\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
 & \quad \left. \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / \\
 & \quad \left. \left(8\sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) / \left((a + b)^3 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^3 \right. \\
 & \quad \left. \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left. \operatorname{Sinh}[dx] \right) / \\
 & \left(12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \right. \\
 & \quad \left. \operatorname{Csch}[c + dx] \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left. (-a \operatorname{Sinh}[dx] + 2b \operatorname{Sinh}[dx]) \right) / \\
 & \left(6(a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^4 \right. \\
 & \quad \left. (a b \operatorname{Sinh}[2c] + 2b^2 \operatorname{Sinh}[2c] - a b \operatorname{Sinh}[2dx]) \right) / \\
 & \left(8(a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right)
 \end{aligned}$$

Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + dx]^4}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 242 leaves, 8 steps):

$$\frac{3 (a^2 + 12 a b + 16 b^2) x}{8 a^5} - \frac{3 \sqrt{b} (5 a^2 + 20 a b + 16 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^5 \sqrt{a+b} d} -$$

$$\frac{(5 a + 8 b) \operatorname{Cosh}[c+d x] \operatorname{Sinh}[c+d x]}{8 a^2 d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} + \frac{\operatorname{Cosh}[c+d x]^3 \operatorname{Sinh}[c+d x]}{4 a d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} -$$

$$\frac{b (7 a + 12 b) \operatorname{Tanh}[c+d x]}{8 a^3 d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{3 b (a+2 b) \operatorname{Tanh}[c+d x]}{2 a^4 d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 4019 leaves):

$$\left(3 (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \left(\frac{(3 a^2+8 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \right. \right.$$

$$\left. \left. \frac{(a \sqrt{b} (3 a^2+16 a b+16 b^2+3 a (a+2 b) \operatorname{Cosh}[2(c+d x)]) \operatorname{Sinh}[2(c+d x)])}{((a+b)^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2)} \right) \right) / (16384 b^{5/2} d (a+b \operatorname{Sech}[c+d x]^2)^3) +$$

$$\left((a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \left(- \frac{3 a (a+2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \right. \right.$$

$$\left. \left. \frac{(\sqrt{b} (3 a^3+14 a^2 b+24 a b^2+16 b^3+a (3 a^2+4 a b+4 b^2) \operatorname{Cosh}[2(c+d x)]) \operatorname{Sinh}[2(c+d x)])}{((a+b)^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2)} \right) \right) /$$

$$(16384 b^{5/2} d (a+b \operatorname{Sech}[c+d x]^2)^3) - \frac{1}{512 (a+b \operatorname{Sech}[c+d x]^2)^3}$$

$$3 (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6$$

$$\left(\frac{1}{(a+b)^2} (3 a^5-10 a^4 b+80 a^3 b^2+480 a^2 b^3+640 a b^4+256 b^5) \left(\left(\operatorname{ArcTan}[\operatorname{Sech}[d x]] \right. \right. \right.$$

$$\left. \left. \left(- \frac{\operatorname{Im} \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{\operatorname{Im} \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right) \operatorname{Cosh}[2 c] \right) /$$

$$(64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}) - \left(\operatorname{ArcTan}[\operatorname{Sech}[d x]] \right.$$

$$\left. \left(- \frac{\operatorname{Im} \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{\operatorname{Im} \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right) \operatorname{Sinh}[2 c] \right) /$$

$$\begin{aligned}
 & \left(64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) + \\
 & \frac{1}{128 a^3 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] \\
 & \left(768 a^4 b^2 d x \operatorname{Cosh}[2 c] + 3584 a^3 b^3 d x \operatorname{Cosh}[2 c] + 6912 a^2 b^4 d x \operatorname{Cosh}[2 c] + \right. \\
 & 6144 a b^5 d x \operatorname{Cosh}[2 c] + 2048 b^6 d x \operatorname{Cosh}[2 c] + 512 a^4 b^2 d x \operatorname{Cosh}[2 d x] + \\
 & 2048 a^3 b^3 d x \operatorname{Cosh}[2 d x] + 2560 a^2 b^4 d x \operatorname{Cosh}[2 d x] + 1024 a b^5 d x \operatorname{Cosh}[2 d x] + \\
 & 512 a^4 b^2 d x \operatorname{Cosh}[4 c+2 d x] + 2048 a^3 b^3 d x \operatorname{Cosh}[4 c+2 d x] + \\
 & 2560 a^2 b^4 d x \operatorname{Cosh}[4 c+2 d x] + 1024 a b^5 d x \operatorname{Cosh}[4 c+2 d x] + \\
 & 128 a^4 b^2 d x \operatorname{Cosh}[2 c+4 d x] + 256 a^3 b^3 d x \operatorname{Cosh}[2 c+4 d x] + \\
 & 128 a^2 b^4 d x \operatorname{Cosh}[2 c+4 d x] + 128 a^4 b^2 d x \operatorname{Cosh}[6 c+4 d x] + \\
 & 256 a^3 b^3 d x \operatorname{Cosh}[6 c+4 d x] + 128 a^2 b^4 d x \operatorname{Cosh}[6 c+4 d x] - 9 a^6 \operatorname{Sinh}[2 c] + \\
 & 12 a^5 b \operatorname{Sinh}[2 c] + 684 a^4 b^2 \operatorname{Sinh}[2 c] + 2880 a^3 b^3 \operatorname{Sinh}[2 c] + 5280 a^2 b^4 \operatorname{Sinh}[2 c] + \\
 & 4608 a b^5 \operatorname{Sinh}[2 c] + 1536 b^6 \operatorname{Sinh}[2 c] + 9 a^6 \operatorname{Sinh}[2 d x] - 14 a^5 b \operatorname{Sinh}[2 d x] - \\
 & 608 a^4 b^2 \operatorname{Sinh}[2 d x] - 2112 a^3 b^3 \operatorname{Sinh}[2 d x] - 2560 a^2 b^4 \operatorname{Sinh}[2 d x] - \\
 & 1024 a b^5 \operatorname{Sinh}[2 d x] - 3 a^6 \operatorname{Sinh}[4 c+2 d x] + 10 a^5 b \operatorname{Sinh}[4 c+2 d x] + \\
 & 304 a^4 b^2 \operatorname{Sinh}[4 c+2 d x] + 1056 a^3 b^3 \operatorname{Sinh}[4 c+2 d x] + 1280 a^2 b^4 \operatorname{Sinh}[4 c+2 d x] + \\
 & 512 a b^5 \operatorname{Sinh}[4 c+2 d x] + 3 a^6 \operatorname{Sinh}[2 c+4 d x] - 12 a^5 b \operatorname{Sinh}[2 c+4 d x] - \\
 & \left. 204 a^4 b^2 \operatorname{Sinh}[2 c+4 d x] - 384 a^3 b^3 \operatorname{Sinh}[2 c+4 d x] - 192 a^2 b^4 \operatorname{Sinh}[2 c+4 d x] \right) + \\
 & \frac{1}{512 (a+b \operatorname{Sech}[c+d x])^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
 & \operatorname{Sech}[c+d x]^6 \\
 & \left(\frac{12 (7 a^2+32 a b+32 b^2) x}{a^5} + \right. \\
 & \frac{1}{(a+b)^2} (a^7-14 a^6 b+336 a^5 b^2+5600 a^4 b^3+22400 a^3 b^4+37632 a^2 b^5+28672 a b^6+8192 b^7) \\
 & \left. \left(\left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+ \right. \right. \\
 & \left. \left. a \operatorname{Sinh}[2 c+d x]) \right) \operatorname{Cosh}[2 c] \right) / \left(64 a^5 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) - \\
 & \left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \right. \right. \\
 & \left. \left. \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[\right. \\
 & \left. \left. 2 c+d x] \right) \operatorname{Sinh}[2 c] \right) / \left(64 a^5 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) + \\
 & \frac{1}{16 a^5 b (a+b) d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] (-a^6 \operatorname{Sinh}[2 c]-52 a^5 b \operatorname{Sinh}[2 c]- \\
 & 500 a^4 b^2 \operatorname{Sinh}[2 c]-1920 a^3 b^3 \operatorname{Sinh}[2 c]-3520 a^2 b^4 \operatorname{Sinh}[2 c]-3072 a b^5 \operatorname{Sinh}[2 c]- \\
 & 1024 b^6 \operatorname{Sinh}[2 c]+a^6 \operatorname{Sinh}[2 d x]+50 a^5 b \operatorname{Sinh}[2 d x]+400 a^4 b^2 \operatorname{Sinh}[2 d x]+
 \end{aligned}$$

$$\begin{aligned}
 & 1120 a^3 b^3 \operatorname{Sinh}[2 d x] + 1280 a^2 b^4 \operatorname{Sinh}[2 d x] + 512 a b^5 \operatorname{Sinh}[2 d x] + \\
 & \frac{1}{64 a^5 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])} \operatorname{Sech}[2 c] \\
 & \left(-3 a^7 \operatorname{Sinh}[2 c] + 42 a^6 b \operatorname{Sinh}[2 c] + 2192 a^5 b^2 \operatorname{Sinh}[2 c] + 16480 a^4 b^3 \operatorname{Sinh}[2 c] + \right. \\
 & \quad 51200 a^3 b^4 \operatorname{Sinh}[2 c] + 77824 a^2 b^5 \operatorname{Sinh}[2 c] + 57344 a b^6 \operatorname{Sinh}[2 c] + 16384 b^7 \operatorname{Sinh}[2 c] + \\
 & \quad 3 a^7 \operatorname{Sinh}[2 d x] - 44 a^6 b \operatorname{Sinh}[2 d x] - 1900 a^5 b^2 \operatorname{Sinh}[2 d x] - 10880 a^4 b^3 \operatorname{Sinh}[2 d x] - \\
 & \quad \left. 23360 a^3 b^4 \operatorname{Sinh}[2 d x] - 21504 a^2 b^5 \operatorname{Sinh}[2 d x] - 7168 a b^6 \operatorname{Sinh}[2 d x] \right) + \\
 & (a+2 b) \left(-\frac{12 \operatorname{Cosh}[2 c+2 d x]}{a^4 d} - \frac{12 \operatorname{Sinh}[2 c+2 d x]}{a^4 d} \right) + \\
 & (a+2 b) \left(\frac{12 \operatorname{Cosh}[2 c+2 d x]}{a^4 d} - \frac{12 \operatorname{Sinh}[2 c+2 d x]}{a^4 d} \right) + \\
 & \left. \frac{2 \operatorname{Sinh}[4 c+4 d x]}{a^3 d} \right) + \\
 & \frac{1}{256 (a+b \operatorname{Sech}[c+d x])^2} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
 & \operatorname{Sech}[c+d x]^6 \\
 & \left(\frac{1}{(a+b)^2} (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \\
 & \left. \left(-\left(\left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[\right. \right. \\
 & \quad \left. \left. 2 c+d x]) \right) \operatorname{Cosh}[2 c] \right) / \left(64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) \right) + \\
 & \left(3 i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \right. \right. \\
 & \quad \left. \left. \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[\right. \right. \\
 & \quad \left. \left. 2 c+d x]) \right) \operatorname{Sinh}[2 c] \right) / \left(64 a^4 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) \right) + \\
 & \frac{1}{128 a^4 b^2 (a+b)^2 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^2} \operatorname{Sech}[2 c] (-4608 a^5 b^2 d x \operatorname{Cosh}[2 c] - \\
 & \quad 30720 a^4 b^3 d x \operatorname{Cosh}[2 c] - 84480 a^3 b^4 d x \operatorname{Cosh}[2 c] - 119808 a^2 b^5 d x \operatorname{Cosh}[2 c] - \\
 & \quad 86016 a b^6 d x \operatorname{Cosh}[2 c] - 24576 b^7 d x \operatorname{Cosh}[2 c] - 3072 a^5 b^2 d x \operatorname{Cosh}[2 d x] - \\
 & \quad 18432 a^4 b^3 d x \operatorname{Cosh}[2 d x] - 39936 a^3 b^4 d x \operatorname{Cosh}[2 d x] - 36864 a^2 b^5 d x \operatorname{Cosh}[2 d x] - \\
 & \quad 12288 a b^6 d x \operatorname{Cosh}[2 d x] - 3072 a^5 b^2 d x \operatorname{Cosh}[4 c+2 d x] - \\
 & \quad 18432 a^4 b^3 d x \operatorname{Cosh}[4 c+2 d x] - 39936 a^3 b^4 d x \operatorname{Cosh}[4 c+2 d x] - \\
 & \quad 36864 a^2 b^5 d x \operatorname{Cosh}[4 c+2 d x] - 12288 a b^6 d x \operatorname{Cosh}[4 c+2 d x] - \\
 & \quad 768 a^5 b^2 d x \operatorname{Cosh}[2 c+4 d x] - 3072 a^4 b^3 d x \operatorname{Cosh}[2 c+4 d x] - \\
 & \quad 3840 a^3 b^4 d x \operatorname{Cosh}[2 c+4 d x] - 1536 a^2 b^5 d x \operatorname{Cosh}[2 c+4 d x] - \\
 & \quad 768 a^5 b^2 d x \operatorname{Cosh}[6 c+4 d x] - 3072 a^4 b^3 d x \operatorname{Cosh}[6 c+4 d x] - \\
 & \quad 3840 a^3 b^4 d x \operatorname{Cosh}[6 c+4 d x] - 1536 a^2 b^5 d x \operatorname{Cosh}[6 c+4 d x] + 9 a^7 \operatorname{Sinh}[2 c] - \\
 & \quad 54 a^6 b \operatorname{Sinh}[2 c] - 2392 a^5 b^2 \operatorname{Sinh}[2 c] - 13968 a^4 b^3 \operatorname{Sinh}[2 c] -
 \end{aligned}$$

$$\begin{aligned}
 & 36480 a^3 b^4 \operatorname{Sinh}[2 c] - 50432 a^2 b^5 \operatorname{Sinh}[2 c] - 35840 a b^6 \operatorname{Sinh}[2 c] - \\
 & 10240 b^7 \operatorname{Sinh}[2 c] - 9 a^7 \operatorname{Sinh}[2 d x] + 56 a^6 b \operatorname{Sinh}[2 d x] + 2552 a^5 b^2 \operatorname{Sinh}[2 d x] + \\
 & 13184 a^4 b^3 \operatorname{Sinh}[2 d x] + 27072 a^3 b^4 \operatorname{Sinh}[2 d x] + 24576 a^2 b^5 \operatorname{Sinh}[2 d x] + \\
 & 8192 a b^6 \operatorname{Sinh}[2 d x] + 3 a^7 \operatorname{Sinh}[4 c + 2 d x] - 24 a^6 b \operatorname{Sinh}[4 c + 2 d x] - \\
 & 600 a^5 b^2 \operatorname{Sinh}[4 c + 2 d x] - 3200 a^4 b^3 \operatorname{Sinh}[4 c + 2 d x] - 6720 a^3 b^4 \operatorname{Sinh}[4 c + 2 d x] - \\
 & 6144 a^2 b^5 \operatorname{Sinh}[4 c + 2 d x] - 2048 a b^6 \operatorname{Sinh}[4 c + 2 d x] - 3 a^7 \operatorname{Sinh}[2 c + 4 d x] + \\
 & 26 a^6 b \operatorname{Sinh}[2 c + 4 d x] + 992 a^5 b^2 \operatorname{Sinh}[2 c + 4 d x] + 3648 a^4 b^3 \operatorname{Sinh}[2 c + 4 d x] + \\
 & 4480 a^3 b^4 \operatorname{Sinh}[2 c + 4 d x] + 1792 a^2 b^5 \operatorname{Sinh}[2 c + 4 d x] + 256 a^5 b^2 \operatorname{Sinh}[6 c + 4 d x] + \\
 & 1024 a^4 b^3 \operatorname{Sinh}[6 c + 4 d x] + 1280 a^3 b^4 \operatorname{Sinh}[6 c + 4 d x] + 512 a^2 b^5 \operatorname{Sinh}[6 c + 4 d x] + \\
 & 64 a^5 b^2 \operatorname{Sinh}[4 c + 6 d x] + 128 a^4 b^3 \operatorname{Sinh}[4 c + 6 d x] + 64 a^3 b^4 \operatorname{Sinh}[4 c + 6 d x] + \\
 & 64 a^5 b^2 \operatorname{Sinh}[8 c + 6 d x] + 128 a^4 b^3 \operatorname{Sinh}[8 c + 6 d x] + 64 a^3 b^4 \operatorname{Sinh}[8 c + 6 d x] \Big) - \\
 & \frac{1}{8192 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
 & \operatorname{Sech}[c+d x]^6 \\
 & \left(\left(6 a^2 \operatorname{ArcTanh} \left[\left(\operatorname{Sech}[d x] \left(\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c] \right) \left((a+2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c+d x] \right) \right] \right) \right) / \right. \\
 & \quad \left. \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) \right) \\
 & \quad \left(\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c] \right) \Big) / \left(\sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) + \\
 & \quad \left(a \operatorname{Sech}[2 c] \left((-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \operatorname{Sinh}[2 d x] + \right. \right. \\
 & \quad \left. \left. a \left(-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3 \right) \operatorname{Sinh}[2(c+2 d x)] + \right. \right. \\
 & \quad \left. \left. \left(3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4 \right) \operatorname{Sinh}[4 c+2 d x] \right) + \right. \\
 & \quad \left. \left(9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5 \right) \operatorname{Tanh}[2 c] \right) / \\
 & \quad \left(a^2 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \right) \Big)
 \end{aligned}$$

Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c+d x]^3}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{5 \sqrt{b} (3 a+7 b) \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Cosh}[c+d x]}{\sqrt{b}} \right]}{8 a^{9/2} d} - \frac{(a+3 b) \operatorname{Cosh}[c+d x]}{a^4 d} + \\
 \frac{\operatorname{Cosh}[c+d x]^3}{3 a^3 d} + \frac{b^2 (a+b) \operatorname{Cosh}[c+d x]}{4 a^4 d (b+a \operatorname{Cosh}[c+d x]^2)^2} - \frac{b (9 a+13 b) \operatorname{Cosh}[c+d x]}{8 a^4 d (b+a \operatorname{Cosh}[c+d x]^2)}$$

Result (type 3, 1364 leaves):

$$- \left(\left(3 \left(\frac{1}{\sqrt{a}} 3 \left(\operatorname{ArcTan} \left[\frac{\sqrt{a} - i \sqrt{a+b} \operatorname{Tanh} \left[\frac{1}{2} (c+d x) \right]}{\sqrt{b}} \right] \right) + \right. \right. \right.$$

$$\begin{aligned}
 & \left. \left(\frac{\text{ArcTan}\left[\frac{\sqrt{a} + i\sqrt{a+b} \tanh\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right]}{\sqrt{b}}\right) + \right. \\
 & \left. \frac{2\sqrt{b} \cosh[c+dx] (3a+10b+3a \cosh[2(c+dx)])}{(a+2b+a \cosh[2(c+dx)])^2} \right) (a+2b+a \cosh[2c+2dx])^3 \\
 & \left. \text{Sech}[c+dx]^6 \right) / (8192 b^{5/2} d (a+b \text{Sech}[c+dx]^2)^3) - \\
 & \frac{1}{2048 a^{3/2} b^{5/2} d (a+b \text{Sech}[c+dx]^2)^3} \\
 & \left(- (3a-4b) \left(\text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2}) \sinh[c] \tanh\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
 & \left. \left. \left. \cosh[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2} \tanh\left[\frac{dx}{2}\right] \right) \right] \right) \right) + \right. \\
 & \left. \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2}) \sinh[c] \tanh\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
 & \left. \left. \left. \cosh[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2} \tanh\left[\frac{dx}{2}\right] \right) \right] \right) \right) - \right. \\
 & \left. \left(2\sqrt{a}\sqrt{b} \cosh[c+dx] (3a^2+6ab+8b^2+a(3a-4b) \cosh[2(c+dx)]) \right) / \right. \\
 & \left. (a+2b+a \cosh[2(c+dx)])^2 \right) \\
 & (a+2b+a \cosh[2c+2dx])^3 \text{Sech}[c+dx]^6 + \\
 & \frac{1}{49152 a^{9/2} b^{5/2} d (a+b \text{Sech}[c+dx]^2)^3} \\
 & \left(3(3a^4-40a^3b+720a^2b^2+6720ab^3+8960b^4) \right. \\
 & \left. \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2}) \sinh[c] \tanh\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
 & \left. \left. \left. \cosh[c] \left(\sqrt{a} - i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2} \tanh\left[\frac{dx}{2}\right] \right) \right] \right) \right) + \right. \\
 & \left. 3(3a^4-40a^3b+720a^2b^2+6720ab^3+8960b^4) \text{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\
 & \left. \left. \left((\sqrt{a} + i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2}) \sinh[c] \tanh\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
 & \left. \left. \left. \cosh[c] \left(\sqrt{a} + i\sqrt{a+b} \sqrt{(\cosh[c]-\sinh[c])^2} \tanh\left[\frac{dx}{2}\right] \right) \right] \right) \right) + \right. \\
 & \left. \left(2\sqrt{a}\sqrt{b} \cosh[c+dx] (9a^5-90a^4b-10144a^3b^2-48672a^2b^3-85120ab^4- \right. \right. \\
 & \left. \left. 53760b^5+a(9a^4-120a^3b-12432a^2b^2-47936ab^3-44800b^4) \cosh[2(c+dx)] - \right. \right. \\
 & \left. \left. 128a^2b^2(15a+28b) \cosh[4(c+dx)] + 128a^3b^2 \cosh[6(c+dx)]) \right) / \right. \\
 & \left. (a+2b+a \cosh[2(c+dx)])^2 \right) (a+2b+a \cosh[2c+2dx])^3 \text{Sech}[c+dx]^6 +
 \end{aligned}$$

$$\frac{1}{16384 a^{7/2} d (a + b \operatorname{Sech}[c + d x]^2)^3} \operatorname{Sech}[c + d x]^6 \left(-\frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \right. \right. \right. \right. \\ \left. \left. \left. \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) - \\ \frac{1}{b^{5/2}} 3 (a^3 - 8 a^2 b + 80 a b^2 + 320 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \right. \right. \right. \\ \left. \left. \left. \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) + \\ 512 \sqrt{a} \operatorname{Cosh}[c] \operatorname{Cosh}[d x] - \frac{8 \sqrt{a} (a^3 + 24 a^2 b + 80 a b^2 + 64 b^3) \operatorname{Cosh}[c + d x]}{b (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2} - \\ \frac{2 \sqrt{a} (3 a^3 - 24 a^2 b - 400 a b^2 - 576 b^3) \operatorname{Cosh}[c + d x]}{b^2 (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])} + 512 \sqrt{a} \operatorname{Sinh}[c] \operatorname{Sinh}[d x] \Bigg)$$

Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sinh}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{(a + 6 b) x}{2 a^4} + \frac{\sqrt{b} (15 a^2 + 40 a b + 24 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{8 a^4 (a + b)^{3/2} d} + \frac{\operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]}{2 a d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} + \\ \frac{3 b \operatorname{Tanh}[c + d x]}{4 a^2 d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} + \frac{b (11 a + 12 b) \operatorname{Tanh}[c + d x]}{8 a^3 (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 3106 leaves):

$$-\left(\left(5 (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6 \left(\frac{(3 a^2 + 8 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} - \right. \right. \right. \\ \left. \left. \left. (a \sqrt{b} (3 a^2 + 16 a b + 16 b^2 + 3 a (a + 2 b) \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sinh}[2 (c + d x)]) \right) / \right. \right. \\ \left. \left. \left. \left((a + b)^2 (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \right) \right) \right) / (8192 b^{5/2} d (a + b \operatorname{Sech}[c + d x]^2)^3) \right) - \\ \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6 \left(-\frac{3 a (a + 2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} + \right. \right.$$

$$\begin{aligned}
 & \left(\sqrt{b} \left(3 a^3 + 14 a^2 b + 24 a b^2 + 16 b^3 + a \left(3 a^2 + 4 a b + 4 b^2 \right) \operatorname{Cosh} \left[2 \left(c + d x \right) \right] \right) \right. \\
 & \left. \operatorname{Sinh} \left[2 \left(c + d x \right) \right] \right) / \left(\left(a + b \right)^2 \left(a + 2 b + a \operatorname{Cosh} \left[2 \left(c + d x \right) \right] \right)^2 \right) \Bigg) / \\
 & \left(2048 b^{5/2} d \left(a + b \operatorname{Sech} \left[c + d x \right]^2 \right)^3 \right) + \frac{1}{32 \left(a + b \operatorname{Sech} \left[c + d x \right]^2 \right)^3} \\
 & \left(a + 2 b + a \operatorname{Cosh} \left[2 c + 2 d x \right] \right)^3 \\
 & \operatorname{Sech} \left[c + d x \right]^6 \\
 & \left(\frac{1}{\left(a + b \right)^2} \left(3 a^5 - 10 a^4 b + 80 a^3 b^2 + 480 a^2 b^3 + 640 a b^4 + 256 b^5 \right) \left(\left(i \operatorname{ArcTan} \left[\operatorname{Sech} \left[d x \right] \right. \right. \right. \right. \\
 & \left. \left. \left. \left(- \frac{i \operatorname{Cosh} \left[2 c \right]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]}} + \frac{i \operatorname{Sinh} \left[2 c \right]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]}} \right) \right. \right. \right. \right. \\
 & \left. \left. \left. \left(- a \operatorname{Sinh} \left[d x \right] - 2 b \operatorname{Sinh} \left[d x \right] + a \operatorname{Sinh} \left[2 c + d x \right] \right) \right) \operatorname{Cosh} \left[2 c \right] \right) \right) / \\
 & \left(64 a^3 b^2 \sqrt{a + b} d \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]} \right) - \left(i \operatorname{ArcTan} \left[\operatorname{Sech} \left[d x \right] \right. \right. \\
 & \left. \left. \left(- \frac{i \operatorname{Cosh} \left[2 c \right]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]}} + \frac{i \operatorname{Sinh} \left[2 c \right]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]}} \right) \right. \right. \\
 & \left. \left. \left(- a \operatorname{Sinh} \left[d x \right] - 2 b \operatorname{Sinh} \left[d x \right] + a \operatorname{Sinh} \left[2 c + d x \right] \right) \right) \operatorname{Sinh} \left[2 c \right] \right) / \\
 & \left. \left(64 a^3 b^2 \sqrt{a + b} d \sqrt{b \operatorname{Cosh} \left[4 c \right] - b \operatorname{Sinh} \left[4 c \right]} \right) \right) + \\
 & \frac{1}{128 a^3 b^2 \left(a + b \right)^2 d \left(a + 2 b + a \operatorname{Cosh} \left[2 c + 2 d x \right] \right)^2} \operatorname{Sech} \left[2 c \right] \\
 & \left(768 a^4 b^2 d x \operatorname{Cosh} \left[2 c \right] + 3584 a^3 b^3 d x \operatorname{Cosh} \left[2 c \right] + 6912 a^2 b^4 d x \operatorname{Cosh} \left[2 c \right] + \right. \\
 & 6144 a b^5 d x \operatorname{Cosh} \left[2 c \right] + 2048 b^6 d x \operatorname{Cosh} \left[2 c \right] + 512 a^4 b^2 d x \operatorname{Cosh} \left[2 d x \right] + \\
 & 2048 a^3 b^3 d x \operatorname{Cosh} \left[2 d x \right] + 2560 a^2 b^4 d x \operatorname{Cosh} \left[2 d x \right] + 1024 a b^5 d x \operatorname{Cosh} \left[2 d x \right] + \\
 & 512 a^4 b^2 d x \operatorname{Cosh} \left[4 c + 2 d x \right] + 2048 a^3 b^3 d x \operatorname{Cosh} \left[4 c + 2 d x \right] + \\
 & 2560 a^2 b^4 d x \operatorname{Cosh} \left[4 c + 2 d x \right] + 1024 a b^5 d x \operatorname{Cosh} \left[4 c + 2 d x \right] + \\
 & 128 a^4 b^2 d x \operatorname{Cosh} \left[2 c + 4 d x \right] + 256 a^3 b^3 d x \operatorname{Cosh} \left[2 c + 4 d x \right] + \\
 & 128 a^2 b^4 d x \operatorname{Cosh} \left[2 c + 4 d x \right] + 128 a^4 b^2 d x \operatorname{Cosh} \left[6 c + 4 d x \right] + \\
 & 256 a^3 b^3 d x \operatorname{Cosh} \left[6 c + 4 d x \right] + 128 a^2 b^4 d x \operatorname{Cosh} \left[6 c + 4 d x \right] - 9 a^6 \operatorname{Sinh} \left[2 c \right] + \\
 & 12 a^5 b \operatorname{Sinh} \left[2 c \right] + 684 a^4 b^2 \operatorname{Sinh} \left[2 c \right] + 2880 a^3 b^3 \operatorname{Sinh} \left[2 c \right] + 5280 a^2 b^4 \operatorname{Sinh} \left[2 c \right] + \\
 & 4608 a b^5 \operatorname{Sinh} \left[2 c \right] + 1536 b^6 \operatorname{Sinh} \left[2 c \right] + 9 a^6 \operatorname{Sinh} \left[2 d x \right] - 14 a^5 b \operatorname{Sinh} \left[2 d x \right] - \\
 & 608 a^4 b^2 \operatorname{Sinh} \left[2 d x \right] - 2112 a^3 b^3 \operatorname{Sinh} \left[2 d x \right] - 2560 a^2 b^4 \operatorname{Sinh} \left[2 d x \right] - \\
 & 1024 a b^5 \operatorname{Sinh} \left[2 d x \right] - 3 a^6 \operatorname{Sinh} \left[4 c + 2 d x \right] + 10 a^5 b \operatorname{Sinh} \left[4 c + 2 d x \right] + \\
 & 304 a^4 b^2 \operatorname{Sinh} \left[4 c + 2 d x \right] + 1056 a^3 b^3 \operatorname{Sinh} \left[4 c + 2 d x \right] + 1280 a^2 b^4 \operatorname{Sinh} \left[4 c + 2 d x \right] + \\
 & 512 a b^5 \operatorname{Sinh} \left[4 c + 2 d x \right] + 3 a^6 \operatorname{Sinh} \left[2 c + 4 d x \right] - 12 a^5 b \operatorname{Sinh} \left[2 c + 4 d x \right] - \\
 & \left. 204 a^4 b^2 \operatorname{Sinh} \left[2 c + 4 d x \right] - 384 a^3 b^3 \operatorname{Sinh} \left[2 c + 4 d x \right] - 192 a^2 b^4 \operatorname{Sinh} \left[2 c + 4 d x \right] \right) + \\
 & \frac{1}{128 \left(a + b \operatorname{Sech} \left[c + d x \right]^2 \right)^3} \left(a + 2 b + a \operatorname{Cosh} \left[2 c + 2 d x \right] \right)^3
 \end{aligned}$$

$$\begin{aligned}
 & \text{Sech}[c + d x]^6 \\
 & \left(\frac{1}{(a + b)^2} (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \\
 & \quad \left(- \left(\left(3 i \text{ArcTan}[\text{Sech}[d x] \left(- \frac{i \text{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]}} + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{i \text{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]}} \right) (-a \text{Sinh}[d x] - 2 b \text{Sinh}[d x] + a \text{Sinh}[\right. \right. \\
 & \quad \left. \left. 2 c + d x] \right) \right) \text{Cosh}[2 c] \right) / \left(64 a^4 b^2 \sqrt{a + b} d \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]} \right) \Bigg) + \\
 & \quad \left(3 i \text{ArcTan}[\text{Sech}[d x] \left(- \frac{i \text{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]}} + \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{i \text{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]}} \right) (-a \text{Sinh}[d x] - 2 b \text{Sinh}[d x] + a \text{Sinh}[\right. \right. \\
 & \quad \left. \left. 2 c + d x] \right) \right) \text{Sinh}[2 c] \Bigg) / \left(64 a^4 b^2 \sqrt{a + b} d \sqrt{b \text{Cosh}[4 c] - b \text{Sinh}[4 c]} \right) \Bigg) + \\
 & \frac{1}{128 a^4 b^2 (a + b)^2 d (a + 2 b + a \text{Cosh}[2 c + 2 d x])^2} \text{Sech}[2 c] (-4608 a^5 b^2 d x \text{Cosh}[2 c] - \\
 & 30720 a^4 b^3 d x \text{Cosh}[2 c] - 84480 a^3 b^4 d x \text{Cosh}[2 c] - 119808 a^2 b^5 d x \text{Cosh}[2 c] - \\
 & 86016 a b^6 d x \text{Cosh}[2 c] - 24576 b^7 d x \text{Cosh}[2 c] - 3072 a^5 b^2 d x \text{Cosh}[2 d x] - \\
 & 18432 a^4 b^3 d x \text{Cosh}[2 d x] - 39936 a^3 b^4 d x \text{Cosh}[2 d x] - 36864 a^2 b^5 d x \text{Cosh}[2 d x] - \\
 & 12288 a b^6 d x \text{Cosh}[2 d x] - 3072 a^5 b^2 d x \text{Cosh}[4 c + 2 d x] - \\
 & 18432 a^4 b^3 d x \text{Cosh}[4 c + 2 d x] - 39936 a^3 b^4 d x \text{Cosh}[4 c + 2 d x] - \\
 & 36864 a^2 b^5 d x \text{Cosh}[4 c + 2 d x] - 12288 a b^6 d x \text{Cosh}[4 c + 2 d x] - \\
 & 768 a^5 b^2 d x \text{Cosh}[2 c + 4 d x] - 3072 a^4 b^3 d x \text{Cosh}[2 c + 4 d x] - \\
 & 3840 a^3 b^4 d x \text{Cosh}[2 c + 4 d x] - 1536 a^2 b^5 d x \text{Cosh}[2 c + 4 d x] - \\
 & 768 a^5 b^2 d x \text{Cosh}[6 c + 4 d x] - 3072 a^4 b^3 d x \text{Cosh}[6 c + 4 d x] - \\
 & 3840 a^3 b^4 d x \text{Cosh}[6 c + 4 d x] - 1536 a^2 b^5 d x \text{Cosh}[6 c + 4 d x] + 9 a^7 \text{Sinh}[2 c] - \\
 & 54 a^6 b \text{Sinh}[2 c] - 2392 a^5 b^2 \text{Sinh}[2 c] - 13968 a^4 b^3 \text{Sinh}[2 c] - \\
 & 36480 a^3 b^4 \text{Sinh}[2 c] - 50432 a^2 b^5 \text{Sinh}[2 c] - 35840 a b^6 \text{Sinh}[2 c] - \\
 & 10240 b^7 \text{Sinh}[2 c] - 9 a^7 \text{Sinh}[2 d x] + 56 a^6 b \text{Sinh}[2 d x] + 2552 a^5 b^2 \text{Sinh}[2 d x] + \\
 & 13184 a^4 b^3 \text{Sinh}[2 d x] + 27072 a^3 b^4 \text{Sinh}[2 d x] + 24576 a^2 b^5 \text{Sinh}[2 d x] + \\
 & 8192 a b^6 \text{Sinh}[2 d x] + 3 a^7 \text{Sinh}[4 c + 2 d x] - 24 a^6 b \text{Sinh}[4 c + 2 d x] - \\
 & 600 a^5 b^2 \text{Sinh}[4 c + 2 d x] - 3200 a^4 b^3 \text{Sinh}[4 c + 2 d x] - 6720 a^3 b^4 \text{Sinh}[4 c + 2 d x] - \\
 & 6144 a^2 b^5 \text{Sinh}[4 c + 2 d x] - 2048 a b^6 \text{Sinh}[4 c + 2 d x] - 3 a^7 \text{Sinh}[2 c + 4 d x] + \\
 & 26 a^6 b \text{Sinh}[2 c + 4 d x] + 992 a^5 b^2 \text{Sinh}[2 c + 4 d x] + 3648 a^4 b^3 \text{Sinh}[2 c + 4 d x] + \\
 & 4480 a^3 b^4 \text{Sinh}[2 c + 4 d x] + 1792 a^2 b^5 \text{Sinh}[2 c + 4 d x] + 256 a^5 b^2 \text{Sinh}[6 c + 4 d x] + \\
 & 1024 a^4 b^3 \text{Sinh}[6 c + 4 d x] + 1280 a^3 b^4 \text{Sinh}[6 c + 4 d x] + 512 a^2 b^5 \text{Sinh}[6 c + 4 d x] + \\
 & 64 a^5 b^2 \text{Sinh}[4 c + 6 d x] + 128 a^4 b^3 \text{Sinh}[4 c + 6 d x] + 64 a^3 b^4 \text{Sinh}[4 c + 6 d x] + \\
 & 64 a^5 b^2 \text{Sinh}[8 c + 6 d x] + 128 a^4 b^3 \text{Sinh}[8 c + 6 d x] + 64 a^3 b^4 \text{Sinh}[8 c + 6 d x] \Bigg) + \\
 & \frac{1}{4096 b^2 (a + b)^2 d (a + b \text{Sech}[c + d x]^2)^3} (a + 2 b + a \text{Cosh}[2 c + 2 d x])^3 \\
 & \text{Sech}[c + d x]^6 \\
 & \left(\left(6 a^2 \text{ArcTanh}[(\text{Sech}[d x] (\text{Cosh}[2 c] - \text{Sinh}[2 c]) ((a + 2 b) \text{Sinh}[d x] - a \text{Sinh}[2 c + d x]))] \right) / \right.
 \end{aligned}$$

$$\begin{aligned} & \left(2 \sqrt{a+b} \sqrt{b (\text{Cosh}[c] - \text{Sinh}[c])^4} \right) \\ & \left(\text{Cosh}[2c] - \text{Sinh}[2c] \right) \Big/ \left(\sqrt{a+b} \sqrt{b (\text{Cosh}[c] - \text{Sinh}[c])^4} \right) + \\ & \left(a \text{Sech}[2c] \left((-9a^4 - 16a^3b + 48a^2b^2 + 128ab^3 + 64b^4) \text{Sinh}[2dx] + \right. \right. \\ & \quad \left. \left. a (-3a^3 + 2a^2b + 24ab^2 + 16b^3) \text{Sinh}[2(c+2dx)] + \right. \right. \\ & \quad \left. \left. (3a^4 - 64a^2b^2 - 128ab^3 - 64b^4) \text{Sinh}[4c+2dx] \right) + \right. \\ & \quad \left. (9a^5 + 18a^4b - 64a^3b^2 - 256a^2b^3 - 320ab^4 - 128b^5) \text{Tanh}[2c] \right) \Big/ \\ & \left(a^2 (a + 2b + a \text{Cosh}[2(c+dx)])^2 \right) \end{aligned}$$

Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sinh}[c+dx]}{(a+b \text{Sech}[c+dx])^3} dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\begin{aligned} & - \frac{15 \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cosh}[c+dx]}{\sqrt{b}}\right]}{8 a^{7/2} d} + \frac{15 \text{Cosh}[c+dx]}{8 a^3 d} - \\ & \frac{\text{Cosh}[c+dx]^5}{4 a d (b+a \text{Cosh}[c+dx]^2)^2} - \frac{5 \text{Cosh}[c+dx]^3}{8 a^2 d (b+a \text{Cosh}[c+dx]^2)} \end{aligned}$$

Result (type 3, 1272 leaves):

$$\begin{aligned} & \frac{1}{4096 a^{5/2} b^{5/2} d (a+b \text{Sech}[c+dx])^3} \\ & 5 \left(3 (a^2 - 4ab + 16b^2) \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \right. \right. \right. \\ & \quad \left. \left. \left. \text{Tanh}\left[\frac{dx}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) + \\ & 3 (a^2 - 4ab + 16b^2) \text{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2}) \text{Sinh}[c] \right. \right. \\ & \quad \left. \left. \left. \text{Tanh}\left[\frac{dx}{2}\right] + \text{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \text{Tanh}\left[\frac{dx}{2}\right] \right) \right) \right] \right) + \\ & \frac{8 \sqrt{a} b^{3/2} (a^2 + 12ab + 16b^2) \text{Cosh}[c+dx]}{(a+2b+a \text{Cosh}[2(c+dx)])^2} + \frac{2 \sqrt{a} \sqrt{b} (3a^2 - 12ab - 80b^2) \text{Cosh}[c+dx]}{a+2b+a \text{Cosh}[2(c+dx)]} \Big) \\ & (a+2b+a \text{Cosh}[2c+2dx])^3 \text{Sech}[c+dx]^6 + \left(5 \left(\frac{1}{\sqrt{a}} \right. \right. \\ & \left. \left. 3 \left(\text{ArcTan}\left[\frac{\sqrt{a} - i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{\sqrt{a} + i \sqrt{a+b} \text{Tanh}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{b}}\right] \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{2\sqrt{b} \operatorname{Cosh}[c+dx] (3a+10b+3a \operatorname{Cosh}[2(c+dx)])}{(a+2b+a \operatorname{Cosh}[2(c+dx)])^2} \Bigg) \\
& (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \Bigg) / \\
& (4096 b^{5/2} d (a+b \operatorname{Sech}[c+dx]^2)^3) + \frac{1}{4096 a^{3/2} b^{5/2} d (a+b \operatorname{Sech}[c+dx]^2)^3} \\
& 9 \left(-(3a-4b) \left(\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a}-i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a}-i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) \right) + \right. \\
& \quad \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a}+i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a}+i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) \right) \right) - \\
& \left(2\sqrt{a} \sqrt{b} \operatorname{Cosh}[c+dx] (3a^2+6ab+8b^2+a(3a-4b) \operatorname{Cosh}[2(c+dx)]) \right) / \\
& \left(a+2b+a \operatorname{Cosh}[2(c+dx)] \right)^2 \Bigg) (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 + \\
& \frac{1}{4096 a^{7/2} d (a+b \operatorname{Sech}[c+dx]^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \\
& \operatorname{Sech}[c+dx]^6 \\
& \left(-\frac{1}{b^{5/2}} 3 (a^3-8a^2b+80ab^2+320b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a}-i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a}-i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) \right) - \\
& \frac{1}{b^{5/2}} 3 (a^3-8a^2b+80ab^2+320b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a}+i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{dx}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a}+i\sqrt{a+b} \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{dx}{2}\right] \right) \right] \right) \right) \Bigg) + \\
& 512 \sqrt{a} \operatorname{Cosh}[c] \operatorname{Cosh}[dx] - \frac{8\sqrt{a} (a^3+24a^2b+80ab^2+64b^3) \operatorname{Cosh}[c+dx]}{b (a+2b+a \operatorname{Cosh}[2(c+dx)])^2} - \\
& \left. \frac{2\sqrt{a} (3a^3-24a^2b-400ab^2-576b^3) \operatorname{Cosh}[c+dx]}{b^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])} + 512 \sqrt{a} \operatorname{Sinh}[c] \operatorname{Sinh}[dx] \right)
\end{aligned}$$

Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+dx]}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+d x]}{\sqrt{b}}\right]}{8 a^{5/2} (a+b)^3 d} - \frac{\operatorname{ArcTanh}[\operatorname{Cosh}[c+d x]]}{(a+b)^3 d} - \frac{b \operatorname{Cosh}[c+d x]^3}{4 a (a+b) d (b+a \operatorname{Cosh}[c+d x]^2)^2} - \frac{b (7 a+3 b) \operatorname{Cosh}[c+d x]}{8 a^2 (a+b)^2 d (b+a \operatorname{Cosh}[c+d x]^2)}$$

Result (type 3, 440 leaves):

$$\frac{1}{64 (a+b)^3 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^5 \left(\frac{8 b^2 (a+b)^2}{a^2} - \frac{2 b (a+b) (9 a+5 b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])}{a^2} + \frac{1}{a^{5/2}} \sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Sech}[c+d x] + \frac{1}{a^{5/2}} \sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Sech}[c+d x] - 8 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sech}[c+d x] + 8 (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sech}[c+d x] \right)$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{15 \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 (a+b)^{7/2} d} - \frac{15 \operatorname{Coth}[c+d x]}{8 (a+b)^3 d} + \frac{\operatorname{Coth}[c+d x]}{4 (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} + \frac{5 \operatorname{Coth}[c+d x]}{8 (a+b)^2 d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 981 leaves):

$$\begin{aligned}
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \right. \\
 & \left(- \left(\left(15 i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(- \frac{i \operatorname{Cosh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{i \operatorname{Sinh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / \\
 & \left(64 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) + \left(15 i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
 & \left(- \frac{i \operatorname{Cosh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \\
 & \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \Big/ \\
 & \left. \left(64 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) \Big/ \\
 & \left((a + b)^3 (a + b \operatorname{Sech}[c + dx]^2)^3 \right) + \frac{1}{512 a^2 (a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^3} \\
 & (a + \\
 & 2b + a \operatorname{Cosh}[2c + 2dx]) \\
 & \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^6 \\
 & (-32 a^4 \operatorname{Sinh}[dx] - 64 a^3 b \operatorname{Sinh}[dx] + \\
 & 22 a^2 b^2 \operatorname{Sinh}[dx] + 80 a b^3 \operatorname{Sinh}[dx] + 16 b^4 \operatorname{Sinh}[dx] + \\
 & 32 a^4 \operatorname{Sinh}[3dx] + 46 a^3 b \operatorname{Sinh}[3dx] - 54 a^2 b^2 \operatorname{Sinh}[3dx] - \\
 & 8 a b^3 \operatorname{Sinh}[3dx] - 48 a^4 \operatorname{Sinh}[2c - dx] - \\
 & 128 a^3 b \operatorname{Sinh}[2c - dx] - 106 a^2 b^2 \operatorname{Sinh}[2c - dx] + \\
 & 80 a b^3 \operatorname{Sinh}[2c - dx] + 16 b^4 \operatorname{Sinh}[2c - dx] + 48 a^4 \operatorname{Sinh}[2c + dx] + \\
 & 146 a^3 b \operatorname{Sinh}[2c + dx] + 182 a^2 b^2 \operatorname{Sinh}[2c + dx] + \\
 & 80 a b^3 \operatorname{Sinh}[2c + dx] + 16 b^4 \operatorname{Sinh}[2c + dx] - 32 a^4 \operatorname{Sinh}[4c + dx] - \\
 & 82 a^3 b \operatorname{Sinh}[4c + dx] - 54 a^2 b^2 \operatorname{Sinh}[4c + dx] - 80 a b^3 \operatorname{Sinh}[4c + dx] - \\
 & 16 b^4 \operatorname{Sinh}[4c + dx] - 8 a^4 \operatorname{Sinh}[2c + 3dx] + 18 a^3 b \operatorname{Sinh}[2c + 3dx] + \\
 & 54 a^2 b^2 \operatorname{Sinh}[2c + 3dx] + 8 a b^3 \operatorname{Sinh}[2c + 3dx] + 32 a^4 \operatorname{Sinh}[4c + 3dx] + \\
 & 73 a^3 b \operatorname{Sinh}[4c + 3dx] + 24 a^2 b^2 \operatorname{Sinh}[4c + 3dx] + \\
 & 8 a b^3 \operatorname{Sinh}[4c + 3dx] - 8 a^4 \operatorname{Sinh}[6c + 3dx] - 9 a^3 b \operatorname{Sinh}[6c + 3dx] - \\
 & 24 a^2 b^2 \operatorname{Sinh}[6c + 3dx] - 8 a b^3 \operatorname{Sinh}[6c + 3dx] + \\
 & 8 a^4 \operatorname{Sinh}[2c + 5dx] - 9 a^3 b \operatorname{Sinh}[2c + 5dx] - 2 a^2 b^2 \operatorname{Sinh}[2c + 5dx] + \\
 & 9 a^3 b \operatorname{Sinh}[4c + 5dx] + 2 a^2 b^2 \operatorname{Sinh}[4c + 5dx] + 8 a^4 \operatorname{Sinh}[6c + 5dx])
 \end{aligned}$$

Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c + dx]^3}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 213 leaves, 7 steps):

$$\begin{aligned}
 & \frac{\sqrt{b} (15 a^2 - 10 a b - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cosh}[c+d x]}{\sqrt{b}}\right]}{8 a^{3/2} (a+b)^4 d} + \\
 & \frac{(a-5 b) \operatorname{ArcTanh}[\operatorname{Cosh}[c+d x]]}{2 (a+b)^4 d} + \frac{(2 a-b) b \operatorname{Cosh}[c+d x]}{4 a (a+b)^2 d (b+a \operatorname{Cosh}[c+d x])^2} - \\
 & \frac{(4 a^2 - 9 a b - b^2) \operatorname{Cosh}[c+d x]}{8 a (a+b)^3 d (b+a \operatorname{Cosh}[c+d x])^2} - \frac{\operatorname{Cosh}[c+d x] \operatorname{Coth}[c+d x]^2}{2 (a+b) d (b+a \operatorname{Cosh}[c+d x])^2}
 \end{aligned}$$

Result (type 3, 524 leaves):

$$\begin{aligned}
 & \frac{1}{64 (a+b)^4 d (a+b \operatorname{Sech}[c+d x])^3} (a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^5 \\
 & \left(-\frac{8 b^2 (a+b)^2}{a} + \frac{2 b (a+b) (9 a+b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])}{a} + \frac{1}{a^{3/2}} \right. \\
 & \quad \left. \sqrt{b} (-15 a^2 + 10 a b + b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Tanh}\left[\frac{d x}{2}\right] + \operatorname{Cosh}[c] \left(\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \right) \\
 & \quad (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Sech}[c+d x] + \frac{1}{a^{3/2}} \sqrt{b} (-15 a^2 + 10 a b + b^2) \\
 & \quad \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}) \operatorname{Sinh}[c] \operatorname{Tanh}\left[\frac{d x}{2}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Cosh}[c] \left(\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}\left[\frac{d x}{2}\right] \right) \right) \right] \\
 & \quad (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Sech}[c+d x] - (a+b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \\
 & \quad \operatorname{Csch}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sech}[c+d x] + \\
 & \quad 4(a-5 b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sech}[c+d x] - \\
 & \quad 4(a-5 b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sech}[c+d x] - \\
 & \quad (a+b) (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \operatorname{Sech}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sech}[c+d x] \Big)
 \end{aligned}$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csch}[c+d x]^4}{(a+b \operatorname{Sech}[c+d x])^3} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{5 (3 a - 4 b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 (a+b)^{9/2} d} + \frac{(a-2 b) \operatorname{Coth}[c+d x]}{(a+b)^4 d} - \frac{\operatorname{Coth}[c+d x]^3}{3 (a+b)^3 d} - \\
 & \frac{a b \operatorname{Tanh}[c+d x]}{4 (a+b)^3 d (a+b-b \operatorname{Tanh}[c+d x])^2} - \frac{(7 a-4 b) b \operatorname{Tanh}[c+d x]}{8 (a+b)^4 d (a+b-b \operatorname{Tanh}[c+d x])^2}
 \end{aligned}$$

Result (type 3, 1228 leaves):

$$\begin{aligned}
 & \left((3a - 4b) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \left(\left(5 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \right. \\
 & \quad \left. \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) \right) / \\
 & \quad \left(64 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(5 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
 & \quad \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right) \\
 & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / \\
 & \quad \left(64 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \left. \right) \left. \right) / \\
 & \left((a+b)^4 (a+b \operatorname{Sech}[c+dx]^2)^3 \right) + \frac{1}{6144 a (a+b)^4 d (a+b \operatorname{Sech}[c+dx]^2)^3} \\
 & (a + \\
 & \quad 2b + a \operatorname{Cosh}[2c + 2dx]) \\
 & \operatorname{CsCh}[c] \operatorname{CsCh}[c+dx]^3 \operatorname{Sech}[2c] \\
 & \operatorname{Sech}[c+dx]^6 \\
 & (-176 a^4 \operatorname{Sinh}[dx] - 488 a^3 b \operatorname{Sinh}[dx] - 252 a^2 b^2 \operatorname{Sinh}[dx] - \\
 & \quad 504 a b^3 \operatorname{Sinh}[dx] - 144 b^4 \operatorname{Sinh}[dx] + 96 a^4 \operatorname{Sinh}[3dx] + \\
 & \quad 71 a^3 b \operatorname{Sinh}[3dx] - 344 a^2 b^2 \operatorname{Sinh}[3dx] + 1208 a b^3 \operatorname{Sinh}[3dx] - \\
 & \quad 48 b^4 \operatorname{Sinh}[3dx] - 224 a^4 \operatorname{Sinh}[2c - dx] - 576 a^3 b \operatorname{Sinh}[2c - dx] - \\
 & \quad 124 a^2 b^2 \operatorname{Sinh}[2c - dx] + 2184 a b^3 \operatorname{Sinh}[2c - dx] - 144 b^4 \operatorname{Sinh}[2c - dx] + \\
 & \quad 224 a^4 \operatorname{Sinh}[2c + dx] + 657 a^3 b \operatorname{Sinh}[2c + dx] + 538 a^2 b^2 \operatorname{Sinh}[2c + dx] - \\
 & \quad 984 a b^3 \operatorname{Sinh}[2c + dx] - 144 b^4 \operatorname{Sinh}[2c + dx] - 176 a^4 \operatorname{Sinh}[4c + dx] - \\
 & \quad 569 a^3 b \operatorname{Sinh}[4c + dx] - 666 a^2 b^2 \operatorname{Sinh}[4c + dx] - 1704 a b^3 \operatorname{Sinh}[4c + dx] + \\
 & \quad 144 b^4 \operatorname{Sinh}[4c + dx] - 48 a^4 \operatorname{Sinh}[2c + 3dx] - 111 a^3 b \operatorname{Sinh}[2c + 3dx] - \\
 & \quad 360 a^2 b^2 \operatorname{Sinh}[2c + 3dx] - 312 a b^3 \operatorname{Sinh}[2c + 3dx] + \\
 & \quad 48 b^4 \operatorname{Sinh}[2c + 3dx] + 96 a^4 \operatorname{Sinh}[4c + 3dx] + 152 a^3 b \operatorname{Sinh}[4c + 3dx] - \\
 & \quad 146 a^2 b^2 \operatorname{Sinh}[4c + 3dx] + 728 a b^3 \operatorname{Sinh}[4c + 3dx] + \\
 & \quad 48 b^4 \operatorname{Sinh}[4c + 3dx] - 48 a^4 \operatorname{Sinh}[6c + 3dx] - 192 a^3 b \operatorname{Sinh}[6c + 3dx] - \\
 & \quad 558 a^2 b^2 \operatorname{Sinh}[6c + 3dx] + 168 a b^3 \operatorname{Sinh}[6c + 3dx] - 48 b^4 \operatorname{Sinh}[6c + 3dx] - \\
 & \quad 16 a^4 \operatorname{Sinh}[2c + 5dx] + 598 a^2 b^2 \operatorname{Sinh}[2c + 5dx] - 48 a b^3 \operatorname{Sinh}[2c + 5dx] - \\
 & \quad 72 a^3 b \operatorname{Sinh}[4c + 5dx] - 150 a^2 b^2 \operatorname{Sinh}[4c + 5dx] + 48 a b^3 \operatorname{Sinh}[4c + 5dx] - \\
 & \quad 16 a^4 \operatorname{Sinh}[6c + 5dx] - 27 a^3 b \operatorname{Sinh}[6c + 5dx] + 388 a^2 b^2 \operatorname{Sinh}[6c + 5dx] - \\
 & \quad 45 a^3 b \operatorname{Sinh}[8c + 5dx] + 60 a^2 b^2 \operatorname{Sinh}[8c + 5dx] - 16 a^4 \operatorname{Sinh}[4c + 7dx] + \\
 & \quad 83 a^3 b \operatorname{Sinh}[4c + 7dx] - 6 a^2 b^2 \operatorname{Sinh}[4c + 7dx] - 27 a^3 b \operatorname{Sinh}[6c + 7dx] + \\
 & \quad 6 a^2 b^2 \operatorname{Sinh}[6c + 7dx] - 16 a^4 \operatorname{Sinh}[8c + 7dx] + 56 a^3 b \operatorname{Sinh}[8c + 7dx])
 \end{aligned}$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c+dx]^2 (a+b \operatorname{Sech}[c+dx]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a+b)^2 \operatorname{Tanh}[c+dx]}{d} - \frac{2b(a+b) \operatorname{Tanh}[c+dx]^3}{3d} + \frac{b^2 \operatorname{Tanh}[c+dx]^5}{5d}$$

Result (type 3, 116 leaves):

$$\frac{a^2 \operatorname{Tanh}[c+dx]}{d} + \frac{4ab \operatorname{Tanh}[c+dx]}{3d} + \frac{8b^2 \operatorname{Tanh}[c+dx]}{15d} + \frac{2ab \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{3d} + \frac{4b^2 \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{15d} + \frac{b^2 \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{5d}$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c+dx]^4 (a+b \operatorname{Sech}[c+dx]^2)^2 dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$\frac{(a+b)^2 \operatorname{Tanh}[c+dx]}{d} - \frac{(a+b)(a+3b) \operatorname{Tanh}[c+dx]^3}{3d} + \frac{b(2a+3b) \operatorname{Tanh}[c+dx]^5}{5d} - \frac{b^2 \operatorname{Tanh}[c+dx]^7}{7d}$$

Result (type 3, 190 leaves):

$$\frac{2a^2 \operatorname{Tanh}[c+dx]}{3d} + \frac{16ab \operatorname{Tanh}[c+dx]}{15d} + \frac{16b^2 \operatorname{Tanh}[c+dx]}{35d} + \frac{a^2 \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{3d} + \frac{8ab \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{15d} + \frac{8b^2 \operatorname{Sech}[c+dx]^2 \operatorname{Tanh}[c+dx]}{35d} + \frac{2ab \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{5d} + \frac{6b^2 \operatorname{Sech}[c+dx]^4 \operatorname{Tanh}[c+dx]}{35d} + \frac{b^2 \operatorname{Sech}[c+dx]^6 \operatorname{Tanh}[c+dx]}{7d}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cosh}[c+dx] (a+b \operatorname{Sech}[c+dx]^2)^3 dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{3b(8a^2+4ab+b^2) \operatorname{ArcTan}[\operatorname{Sinh}[c+dx]]}{8d} + \frac{a^3 \operatorname{Sinh}[c+dx]}{d} + \frac{3b^2(4a+b) \operatorname{Sech}[c+dx] \operatorname{Tanh}[c+dx]}{8d} + \frac{b^3 \operatorname{Sech}[c+dx]^3 \operatorname{Tanh}[c+dx]}{4d}$$

Result (type 3, 189 leaves):

$$\frac{1}{d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} (b + a \operatorname{Cosh}[c + d x]^2)^3 \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^4$$

$$\left(6 b (8 a^2 + 4 a b + b^2) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]\right] \operatorname{Cosh}[c] \operatorname{Cosh}[c + d x]^4 + 2 b^3 \operatorname{Cosh}[c + d x] \right.$$

$$\operatorname{Sinh}[c] + 3 b^2 (4 a + b) \operatorname{Cosh}[c + d x]^3 \operatorname{Sinh}[c] + 4 a^3 \operatorname{Cosh}[d x] \operatorname{Cosh}[c + d x]^4 \operatorname{Sinh}[2 c] +$$

$$\left. 2 b^3 \operatorname{Sinh}[d x] + 3 b^2 (4 a + b) \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[d x] + 8 a^3 \operatorname{Cosh}[c]^2 \operatorname{Cosh}[c + d x]^4 \operatorname{Sinh}[d x] \right)$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{(a + b)^3 \operatorname{Tanh}[c + d x]}{d} - \frac{b (a + b)^2 \operatorname{Tanh}[c + d x]^3}{d} + \frac{3 b^2 (a + b) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^3 \operatorname{Tanh}[c + d x]^7}{7 d}$$

Result (type 3, 319 leaves):

$$\frac{1}{280 d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} \operatorname{Sech}[c] \operatorname{Sech}[c + d x] (a + b \operatorname{Sech}[c + d x]^2)^3$$

$$(140 (5 a^3 + 11 a^2 b + 10 a b^2 + 4 b^3) \operatorname{Sinh}[d x] - 35 a (15 a^2 + 26 a b + 16 b^2) \operatorname{Sinh}[2 c + d x] +$$

$$525 a^3 \operatorname{Sinh}[2 c + 3 d x] + 1260 a^2 b \operatorname{Sinh}[2 c + 3 d x] + 1176 a b^2 \operatorname{Sinh}[2 c + 3 d x] +$$

$$336 b^3 \operatorname{Sinh}[2 c + 3 d x] - 210 a^3 \operatorname{Sinh}[4 c + 3 d x] - 210 a^2 b \operatorname{Sinh}[4 c + 3 d x] +$$

$$210 a^3 \operatorname{Sinh}[4 c + 5 d x] + 490 a^2 b \operatorname{Sinh}[4 c + 5 d x] + 392 a b^2 \operatorname{Sinh}[4 c + 5 d x] +$$

$$112 b^3 \operatorname{Sinh}[4 c + 5 d x] - 35 a^3 \operatorname{Sinh}[6 c + 5 d x] + 35 a^3 \operatorname{Sinh}[6 c + 7 d x] +$$

$$70 a^2 b \operatorname{Sinh}[6 c + 7 d x] + 56 a b^2 \operatorname{Sinh}[6 c + 7 d x] + 16 b^3 \operatorname{Sinh}[6 c + 7 d x])$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + d x]^3 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\frac{(64 a^3 + 144 a^2 b + 120 a b^2 + 35 b^3) \operatorname{ArcTan}[\operatorname{Sinh}[c + d x]]}{128 d} +$$

$$\frac{(64 a^3 + 144 a^2 b + 120 a b^2 + 35 b^3) \operatorname{Sech}[c + d x] \operatorname{Tanh}[c + d x]}{128 d} +$$

$$\frac{b (72 a^2 + 92 a b + 35 b^2) \operatorname{Sech}[c + d x]^3 \operatorname{Tanh}[c + d x]}{192 d} +$$

$$\frac{b (12 a + 7 b) \operatorname{Sech}[c + d x]^5 (a + b + a \operatorname{Sinh}[c + d x]^2) \operatorname{Tanh}[c + d x]}{48 d} +$$

$$\frac{b \operatorname{Sech}[c + d x]^7 (a + b + a \operatorname{Sinh}[c + d x]^2)^2 \operatorname{Tanh}[c + d x]}{8 d}$$

Result (type 3, 629 leaves):

$$\begin{aligned} & \left((64 a^3 + 144 a^2 b + 120 a b^2 + 35 b^3) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Cosh}[c + d x]^6 (a + b \operatorname{Sech}[c + d x]^2)^3 \right) / \\ & \left(8 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \\ & \left(\operatorname{Cosh}[c + d x] \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 (24 a b^2 \operatorname{Sinh}[c] + 7 b^3 \operatorname{Sinh}[c]) \right) / \\ & \left(6 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \left(\operatorname{Cosh}[c + d x]^3 \operatorname{Sech}[c] \right. \\ & \left. (a + b \operatorname{Sech}[c + d x]^2)^3 (144 a^2 b \operatorname{Sinh}[c] + 120 a b^2 \operatorname{Sinh}[c] + 35 b^3 \operatorname{Sinh}[c]) \right) / \\ & \left(24 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \left(\operatorname{Cosh}[c + d x]^5 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 \right. \\ & \left. (64 a^3 \operatorname{Sinh}[c] + 144 a^2 b \operatorname{Sinh}[c] + 120 a b^2 \operatorname{Sinh}[c] + 35 b^3 \operatorname{Sinh}[c]) \right) / \\ & \left(16 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \frac{b^3 \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^3 \operatorname{Sinh}[d x]}{d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3} + \\ & \frac{\operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 (24 a b^2 \operatorname{Sinh}[d x] + 7 b^3 \operatorname{Sinh}[d x])}{6 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3} + \\ & \left(\operatorname{Cosh}[c + d x]^2 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 \right. \\ & \left. (144 a^2 b \operatorname{Sinh}[d x] + 120 a b^2 \operatorname{Sinh}[d x] + 35 b^3 \operatorname{Sinh}[d x]) \right) / \\ & \left(24 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \left(\operatorname{Cosh}[c + d x]^4 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^3 \right. \\ & \left. (64 a^3 \operatorname{Sinh}[d x] + 144 a^2 b \operatorname{Sinh}[d x] + 120 a b^2 \operatorname{Sinh}[d x] + 35 b^3 \operatorname{Sinh}[d x]) \right) / \\ & \left(16 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \right) + \frac{b^3 \operatorname{Sech}[c + d x] (a + b \operatorname{Sech}[c + d x]^2)^3 \operatorname{Tanh}[c]}{d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3} \end{aligned}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sech}[c + d x]^4 (a + b \operatorname{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 108 leaves, 3 steps):

$$\frac{(a + b)^3 \operatorname{Tanh}[c + d x]}{d} - \frac{(a + b)^2 (a + 4 b) \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{3 b (a + b) (a + 2 b) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^2 (3 a + 4 b) \operatorname{Tanh}[c + d x]^7}{7 d} + \frac{b^3 \operatorname{Tanh}[c + d x]^9}{9 d}$$

Result (type 3, 348 leaves):

$$\frac{1}{40320 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^9 (63 (125 a^3 + 324 a^2 b + 312 a b^2 + 128 b^3) \operatorname{Sinh}[d x] - 315 a (17 a^2 + 36 a b + 24 b^2) \operatorname{Sinh}[2 c + d x] + 6825 a^3 \operatorname{Sinh}[2 c + 3 d x] + 18648 a^2 b \operatorname{Sinh}[2 c + 3 d x] + 18144 a b^2 \operatorname{Sinh}[2 c + 3 d x] + 5376 b^3 \operatorname{Sinh}[2 c + 3 d x] - 1995 a^3 \operatorname{Sinh}[4 c + 3 d x] - 2520 a^2 b \operatorname{Sinh}[4 c + 3 d x] + 3465 a^3 \operatorname{Sinh}[4 c + 5 d x] + 9072 a^2 b \operatorname{Sinh}[4 c + 5 d x] + 7776 a b^2 \operatorname{Sinh}[4 c + 5 d x] + 2304 b^3 \operatorname{Sinh}[4 c + 5 d x] - 315 a^3 \operatorname{Sinh}[6 c + 5 d x] + 945 a^3 \operatorname{Sinh}[6 c + 7 d x] + 2268 a^2 b \operatorname{Sinh}[6 c + 7 d x] + 1944 a b^2 \operatorname{Sinh}[6 c + 7 d x] + 576 b^3 \operatorname{Sinh}[6 c + 7 d x] + 105 a^3 \operatorname{Sinh}[8 c + 9 d x] + 252 a^2 b \operatorname{Sinh}[8 c + 9 d x] + 216 a b^2 \operatorname{Sinh}[8 c + 9 d x] + 64 b^3 \operatorname{Sinh}[8 c + 9 d x])$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cosh}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b \text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{a^{3/2} \sqrt{a+b} d} + \frac{\text{Sinh}[c + d x]}{a d}$$

Result (type 3, 147 leaves):

$$\left(b \text{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2 (\text{Cosh}[c] + \text{Sinh}[c])}\right] \text{Cosh}[c] - \right. \\ \left. b \text{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2 (\text{Cosh}[c] + \text{Sinh}[c])}\right] \text{Sinh}[c] + \right. \\ \left. \sqrt{a} \sqrt{a+b} \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2 \text{Sinh}[c + d x]}\right) / \left(a^{3/2} \sqrt{a+b} d \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right)$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{\sqrt{a} \sqrt{a+b} d}$$

Result (type 3, 114 leaves):

$$\left(\text{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \text{Csch}[c + d x] \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2 (\text{Cosh}[c] + \text{Sinh}[c])}\right] \right. \\ \left. (a + 2 b + a \text{Cosh}[2 (c + d x)]) \text{Sech}[c + d x]^2 (-\text{Cosh}[c] + \text{Sinh}[c]) \right) / \\ \left(2 \sqrt{a} \sqrt{a+b} d (a + b \text{Sech}[c + d x]^2) \sqrt{(\text{Cosh}[c] - \text{Sinh}[c])^2} \right)$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sech}[c + d x]^3}{a + b \text{Sech}[c + d x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\text{ArcTan}[\text{Sinh}[c + d x]]}{b d} - \frac{\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{b \sqrt{a+b} d}$$

Result (type 3, 194 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^2 \right. \\ \left. \left(\sqrt{a} \operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c + d x] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}\right] \operatorname{Cosh}[c] + \right. \right. \\ \left. \left. 2 \sqrt{a+b} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} (c + d x)\right]\right] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} - \sqrt{a} \right. \right. \\ \left. \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c + d x] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])}\right] \operatorname{Sinh}[c] \right) \right) \Bigg) / \\ \left(2 b \sqrt{a+b} d (a + b \operatorname{Sech}[c + d x]^2) \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \right)$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^4}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$- \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{b^{3/2} \sqrt{a+b} d} + \frac{\operatorname{Tanh}[c + d x]}{b d}$$

Result (type 3, 182 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^2 \right. \\ \left(a \operatorname{ArcTan}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])) \right] \right) / \\ \left(2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (-\operatorname{Cosh}[2 c] + \operatorname{Sinh}[2 c]) + \\ \sqrt{a+b} \operatorname{Sech}[c] \operatorname{Sech}[c + d x] \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \operatorname{Sinh}[d x] \Bigg) / \\ \left(2 b \sqrt{a+b} d (a + b \operatorname{Sech}[c + d x]^2) \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right)$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^5}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$- \frac{(2 a - b) \operatorname{ArcTan}[\operatorname{Sinh}[c + d x]]}{2 b^2 d} + \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + d x]}{\sqrt{a+b}}\right]}{b^2 \sqrt{a+b} d} + \frac{\operatorname{Sech}[c + d x] \operatorname{Tanh}[c + d x]}{2 b d}$$

Result (type 3, 213 leaves):

$$\frac{1}{4 b^2 \sqrt{a+b} d (a+b \operatorname{Sech}[c+d x]^2) \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2}} \operatorname{Cosh}[c] (a+2 b+a \operatorname{Cosh}[2(c+d x)])$$

$$\operatorname{Sech}[c+d x]^2 \left(b \sqrt{a+b} \operatorname{Sech}[c]^2 \operatorname{Sech}[c+d x]^2 \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \operatorname{Sinh}[d x] + \right.$$

$$2 a^{3/2} \operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c+d x] \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c]+\operatorname{Sinh}[c])\right]$$

$$\left. (-1+\operatorname{Tanh}[c]) - \sqrt{a+b} \operatorname{Sech}[c] \sqrt{(\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^2} \left(2(2 a-b) \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c+d x)\right]\right] - b \operatorname{Sech}[c+d x] \operatorname{Tanh}[c]\right)\right)$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+d x]^6}{a+b \operatorname{Sech}[c+d x]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{b^{5/2} \sqrt{a+b} d} - \frac{(a-b) \operatorname{Tanh}[c+d x]}{b^2 d} - \frac{\operatorname{Tanh}[c+d x]^3}{3 b d}$$

Result (type 3, 214 leaves):

$$\left((a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^2 \right.$$

$$\left. \left(3 a^2 \operatorname{ArcTanh}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c]) ((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x])) \right] \right. \right.$$

$$\left. \left(2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4} \right) \right.$$

$$\left. (\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c]) + \sqrt{a+b} \operatorname{Sech}[c+d x] \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4} \right.$$

$$\left. (\operatorname{Sech}[c] (-3 a+2 b+b \operatorname{Sech}[c+d x]^2) \operatorname{Sinh}[d x]+b \operatorname{Sech}[c+d x] \operatorname{Tanh}[c]) \right) \Big/$$

$$\left(6 b^2 \sqrt{a+b} d (a+b \operatorname{Sech}[c+d x]^2) \sqrt{b (\operatorname{Cosh}[c]-\operatorname{Sinh}[c])^4} \right)$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cosh}[c+d x]}{(a+b \operatorname{Sech}[c+d x]^2)^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{b(4 a+3 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c+d x]}{\sqrt{a+b}}\right]}{2 a^{5/2} (a+b)^{3/2} d} + \frac{\operatorname{Sinh}[c+d x]}{a^2 d} + \frac{b^2 \operatorname{Sinh}[c+d x]}{2 a^2 (a+b) d (a+b+a \operatorname{Sinh}[c+d x]^2)}$$

Result (type 3, 234 leaves):

$$\frac{1}{8 a^{5/2} d (a + b \operatorname{Sech}[c + d x]^2)^2} (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^3$$

$$\left(\left(b (4 a + 3 b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a + b} \operatorname{Csch}[c + d x] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])} \right] \right. \right.$$

$$\left. \left. (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x] (\operatorname{Cosh}[c] - \operatorname{Sinh}[c]) \right) \right) /$$

$$\left((a + b)^{3/2} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + 2 \sqrt{a} \operatorname{Cosh}[d x] \right.$$

$$\left. (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x] \operatorname{Sinh}[c] + \right.$$

$$\left. 2 \sqrt{a} \operatorname{Cosh}[c] (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x] \operatorname{Sinh}[d x] + \frac{2 \sqrt{a} b^2 \operatorname{Tanh}[c + d x]}{a + b} \right)$$

Problem 87: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{2 \sqrt{b} (a + b)^{3/2} d} + \frac{\operatorname{Tanh}[c + d x]}{2 (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 187 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^4 \right.$$

$$\left. \left(\left(\operatorname{ArcTanh}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])) \right] \right) \right. \right.$$

$$\left. \left. \left(2 \sqrt{a + b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \right) \right.$$

$$\left. \left. (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) \right) \right) / \left(\sqrt{a + b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \right.$$

$$\left. \left. \operatorname{Sech}[2 c] \operatorname{Sinh}[2 d x] - \frac{(a + 2 b) \operatorname{Tanh}[2 c]}{a} \right) \right) / \left(8 (a + b) d (a + b \operatorname{Sech}[c + d x]^2)^2 \right)$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^3}{(a + b \operatorname{Sech}[c + d x]^2)^2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{2 \sqrt{a} (a + b)^{3/2} d} + \frac{\operatorname{Sinh}[c + d x]}{2 (a + b) d (a + b + a \operatorname{Sinh}[c + d x]^2)}$$

Result (type 3, 150 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^3 \right. \\ \left. \left(\left(\operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])} \right] \right. \right. \right. \\ \left. \left. \left(a + 2b + a \operatorname{Cosh}[2(c + dx)] \right) \operatorname{Sech}[c + dx] (-\operatorname{Cosh}[c] + \operatorname{Sinh}[c]) \right) \right) \right) / \\ \left(\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + 2 \operatorname{Tanh}[c + dx] \right) \left. \right) / \\ (8(a+b)d(a+b \operatorname{Sech}[c + dx]^2)^2)$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^5}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c + dx]]}{b^2 d} - \frac{\sqrt{a} (2a + 3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + dx]}{\sqrt{a+b}} \right]}{2b^2 (a+b)^{3/2} d} - \frac{a \operatorname{Sinh}[c + dx]}{2b (a+b) d (a + b + a \operatorname{Sinh}[c + dx]^2)}$$

Result (type 3, 282 leaves):

$$\frac{1}{8b^2 (a+b)^{3/2} d (a + b \operatorname{Sech}[c + dx]^2)^2 \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2}} \\ (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^3 \\ \left(\sqrt{a} (2a + 3b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])} \right] \right. \\ \left. \operatorname{Cosh}[c] (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx] - \right. \\ \left. (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx] \right. \\ \left. \left(-4(a+b)^{3/2} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2}(c + dx) \right] \right] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} + \sqrt{a} (2a + 3b) \operatorname{ArcTan}\left[\right. \right. \right. \\ \left. \left. \left. \frac{1}{\sqrt{a}} \sqrt{a+b} \operatorname{Csch}[c + dx] \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} (\operatorname{Cosh}[c] + \operatorname{Sinh}[c])} \right] \operatorname{Sinh}[c] \right) \right) - \\ \left. 2ab \sqrt{a+b} \sqrt{(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^2} \operatorname{Tanh}[c + dx] \right)$$

Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + dx]^6}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$-\frac{a(3a+4b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2b^{5/2}(a+b)^{3/2}d} + \frac{\operatorname{Tanh}[c+dx]}{b^2d} + \frac{a^2\operatorname{Tanh}[c+dx]}{2b^2(a+b)d(a+b-b\operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 483 leaves):

$$\left((3a+4b)(a+2b+a\operatorname{Cosh}[2c+2dx])^2\operatorname{Sech}[c+dx]^4 \left(\left(i a \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\ \left. \left. \left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right. \right. \\ \left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \\ \left(8b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) - \left(i a \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\ \left. \left(-\frac{i\operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} + \frac{i\operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]}} \right) \right. \\ \left. \left. (-a\operatorname{Sinh}[dx]-2b\operatorname{Sinh}[dx]+a\operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \\ \left(8b^2\sqrt{a+b}d\sqrt{b\operatorname{Cosh}[4c]-b\operatorname{Sinh}[4c]} \right) \Big) / \left((a+b)(a+b\operatorname{Sech}[c+dx]^2)^2 \right) + \\ \frac{(a+2b+a\operatorname{Cosh}[2c+2dx])^2\operatorname{Sech}[c]\operatorname{Sech}[c+dx]^5\operatorname{Sinh}[dx]}{4b^2d(a+b\operatorname{Sech}[c+dx]^2)^2} + \\ \left((a+2b+a\operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^4 \right. \\ \left. (-a^2\operatorname{Sinh}[2c]-2ab\operatorname{Sinh}[2c]+a^2\operatorname{Sinh}[2dx]) \right) / \\ \left(8b^2(a+b)d(a+b\operatorname{Sech}[c+dx]^2)^2 \right)$$

Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+dx]^7}{(a+b\operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$-\frac{(4a-b)\operatorname{ArcTan}[\operatorname{Sinh}[c+dx]]}{2b^3d} + \frac{a^{3/2}(4a+5b)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Sinh}[c+dx]}{\sqrt{a+b}}\right]}{2b^3(a+b)^{3/2}d} + \\ \frac{a(2a+b)\operatorname{Sinh}[c+dx]}{2b^2(a+b)d(a+b+a\operatorname{Sinh}[c+dx]^2)} + \frac{\operatorname{Sech}[c+dx]\operatorname{Tanh}[c+dx]}{2bd(a+b+a\operatorname{Sinh}[c+dx]^2)}$$

Result (type 3, 1144 leaves):

$$\begin{aligned}
 & - \left(\left((4a - b) \operatorname{ArcTan} \left[\operatorname{Tanh} \left[\frac{c}{2} + \frac{dx}{2} \right] \right] (a + 2b + a \operatorname{Cosh} [2c + 2dx])^2 \operatorname{Sech} [c + dx]^4 \right) / \right. \\
 & \quad \left. (4b^3 d (a + b \operatorname{Sech} [c + dx]^2)^2) \right) + \\
 & \left(\operatorname{Cosh} \left[\frac{c}{2} \right] (a + 2b + a \operatorname{Cosh} [2c + 2dx])^2 \operatorname{Sech} [c] \operatorname{Sech} [c + dx]^5 \operatorname{Sinh} \left[\frac{c}{2} \right] \right) / \\
 & \quad \left(4b^2 d (a + b \operatorname{Sech} [c + dx]^2)^2 \right) + \\
 & \left((4a^3 + 5a^2 b) (a + 2b + a \operatorname{Cosh} [2c + 2dx])^2 \operatorname{Sech} [c + dx]^4 \right. \\
 & \quad \left. \left(- \left(\left(\operatorname{ArcTan} [\operatorname{Csch} [c + dx]] \left(\frac{\sqrt{a+b} \operatorname{Cosh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{\sqrt{a+b} \operatorname{Sinh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} \right) \right) \right] \right. \\
 & \quad \left. \operatorname{Cosh} [c] \right) / \left(16 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]} \right) \right) + \\
 & \left(\operatorname{ArcTan} [\operatorname{Csch} [c + dx]] \left(\frac{\sqrt{a+b} \operatorname{Cosh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} + \right. \right. \\
 & \quad \left. \left. \frac{\sqrt{a+b} \operatorname{Sinh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} \right) \right) \operatorname{Sinh} [c] \right) / \\
 & \quad \left(16 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]} \right) \left. \right) / \left((a+b) (a + b \operatorname{Sech} [c + dx]^2)^2 \right) + \\
 & \left((4a + 5b) (a + 2b + a \operatorname{Cosh} [2c + 2dx])^2 \operatorname{Sech} [c + dx]^4 \right. \\
 & \quad \left(- \left(\left(a^{3/2} \operatorname{ArcTan} [\operatorname{Csch} [c + dx]] \left(\frac{\sqrt{a+b} \operatorname{Cosh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} + \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{\sqrt{a+b} \operatorname{Sinh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} \right) \right) \right] \right. \\
 & \quad \left. \operatorname{Cosh} [c] \right) / \left(16 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]} \right) \right) + \\
 & \left(a^{3/2} \operatorname{ArcTan} [\operatorname{Csch} [c + dx]] \left(\frac{\sqrt{a+b} \operatorname{Cosh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} + \right. \right. \\
 & \quad \left. \left. \frac{\sqrt{a+b} \operatorname{Sinh} [c] \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]}}{\sqrt{a}} \right) \right) \operatorname{Sinh} [c] \right) / \\
 & \quad \left(16 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh} [2c] - \operatorname{Sinh} [2c]} \right) \left. \right) / \left((a+b) (a + b \operatorname{Sech} [c + dx]^2)^2 \right) + \\
 & \left((4a^3 + 5a^2 b) (a + 2b + a \operatorname{Cosh} [2c + 2dx])^2 \operatorname{Sech} [c + dx]^4 \right.
 \end{aligned}$$

$$\left(\frac{\frac{i \operatorname{Cosh}[c] \operatorname{Log}[a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]]}{32 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]}} - \frac{i \operatorname{Log}[a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]] \operatorname{Sinh}[c]}{32 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]}}}{(a+b) (a+b \operatorname{Sech}[c+d x]^2)^2} + \right. \\ \left. \left(4 a + 5 b \right) (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Sech}[c+d x]^4 \right. \\ \left. \left(- \frac{i a^{3/2} \operatorname{Cosh}[c] \operatorname{Log}[a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]]}{32 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]}} + \frac{i a^{3/2} \operatorname{Log}[a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]] \operatorname{Sinh}[c]}{32 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]}} \right) \right) / \left((a+b) (a+b \operatorname{Sech}[c+d x]^2)^2 \right) + \\ \frac{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Sech}[c] \operatorname{Sech}[c+d x]^6 \operatorname{Sinh}[d x]}{8 b^2 d (a+b \operatorname{Sech}[c+d x]^2)^2} + \\ \frac{a^2 (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]) \operatorname{Sech}[c+d x]^3 \operatorname{Tanh}[c+d x]}{4 b^2 (a+b) d (a+b \operatorname{Sech}[c+d x]^2)^2}$$

Problem 96: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 \sqrt{b} (a+b)^{5/2} d} + \\ \frac{\operatorname{Tanh}[c+d x]}{4 (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} + \frac{3 \operatorname{Tanh}[c+d x]}{8 (a+b)^2 d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 258 leaves):

$$\left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c+d x]^6 \right. \\ \left(\left(3 \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[d x] \left(\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c] \right) \left((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x] \right) \right] \right) \right) / \right. \\ \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \\ \left(\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c] \right) \right) / \left(\sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c] - \operatorname{Sinh}[c] \right)^4} \right) + \\ \frac{4 b (a+b) \operatorname{Sech}[2 c] \left((a + 2 b) \operatorname{Sinh}[2 c] - a \operatorname{Sinh}[2 d x] \right)}{a^2} - \frac{1}{a^2} (a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \\ \operatorname{Sech}[2 c] \left((5 a^2 + 16 a b + 8 b^2) \operatorname{Sinh}[2 c] - a (5 a + 2 b) \operatorname{Sinh}[2 d x] \right) \right) \right) / \\ (64 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)^3)$$

Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^4}{(a + b \operatorname{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{(a + 4 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{8 b^{3/2} (a + b)^{5/2} d} - \frac{a \operatorname{Tanh}[c + d x]}{4 b (a + b) d (a + b - b \operatorname{Tanh}[c + d x]^2)^2} + \frac{(a + 4 b) \operatorname{Tanh}[c + d x]}{8 b (a + b)^2 d (a + b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 507 leaves):

$$\begin{aligned}
 & \left((a + 4 b) (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^3 \operatorname{Sech}[c + d x]^6 \right. \\
 & \quad \left(- \left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \left(- \frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right) \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Cosh}[2 c] \right) / \\
 & \quad \left(64 b \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) + \left(i \operatorname{ArcTan}[\operatorname{Sech}[d x]] \right. \\
 & \quad \left. \left(- \frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]}} \right) \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[d x] - 2 b \operatorname{Sinh}[d x] + a \operatorname{Sinh}[2 c + d x]) \right) \operatorname{Sinh}[2 c] \right) / \\
 & \quad \left. \left(64 b \sqrt{a + b} d \sqrt{b \operatorname{Cosh}[4 c] - b \operatorname{Sinh}[4 c]} \right) \right) / \\
 & \left((a + b)^2 (a + b \operatorname{Sech}[c + d x]^2)^3 \right) + \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x]) \right. \\
 & \quad \operatorname{Sech}[\\
 & \quad \quad 2 c] \operatorname{Sech}[c + d x]^6 \\
 & \quad \left. (-a \operatorname{Sinh}[2 c] - 2 b \operatorname{Sinh}[2 c] + a \operatorname{Sinh}[2 d x]) \right) / \left(16 \right. \\
 & \quad a \\
 & \quad (a + b) \\
 & \quad d \\
 & \quad \left. (a + b \operatorname{Sech}[c + d x]^2)^3 \right) + \\
 & \left((a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^2 \operatorname{Sech}[2 c] \operatorname{Sech}[c + d x]^6 \right. \\
 & \quad \left. (a \operatorname{Sinh}[2 c] + 4 b \operatorname{Sinh}[2 c] - a \operatorname{Sinh}[2 d x] + 2 b \operatorname{Sinh}[2 d x]) \right) / \\
 & \quad \left(64 b (a + b)^2 d (a + b \operatorname{Sech}[c + d x]^2)^3 \right)
 \end{aligned}$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sech}[c + d x]^7}{(a + b \operatorname{Sech}[c + d x]^2)^3} dx$$

Optimal (type 3, 153 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}[\operatorname{Sinh}[c + d x]]}{b^3 d} - \frac{\sqrt{a} (8 a^2 + 20 a b + 15 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Sinh}[c + d x]}{\sqrt{a + b}}\right]}{8 b^3 (a + b)^{5/2} d} - \frac{a \operatorname{Sinh}[c + d x]}{4 b (a + b) d (a + b + a \operatorname{Sinh}[c + d x]^2)^2} - \frac{a (4 a + 7 b) \operatorname{Sinh}[c + d x]}{8 b^2 (a + b)^2 d (a + b + a \operatorname{Sinh}[c + d x]^2)}$$

Result (type 3, 1120 leaves):

$$\begin{aligned}
 & \frac{\text{ArcTan}\left[\text{Tanh}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6}{4b^3 d (a + b \text{Sech}[c + dx]^2)^3} + \\
 & \left((8a^3 + 20a^2b + 15ab^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\
 & \left. \left(\left(\text{ArcTan}[\text{Csch}[c + dx]] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right) \right) \right. \right. \\
 & \left. \left. \text{Cosh}[c] \right) / \left(128 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]} \right) - \right. \\
 & \left. \left(\text{ArcTan}[\text{Csch}[c + dx]] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right) \right) \text{Sinh}[c] \right) / \right. \\
 & \left. \left(128 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]} \right) \right) / \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) + \\
 & \left((8a^2 + 20ab + 15b^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \text{Sech}[c + dx]^6 \right. \\
 & \left. \left(\left(\sqrt{a} \text{ArcTan}[\text{Csch}[c + dx]] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right) \right) \right. \right. \\
 & \left. \left. \text{Cosh}[c] \right) / \left(128 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]} \right) - \right. \\
 & \left. \left(\sqrt{a} \text{ArcTan}[\text{Csch}[c + dx]] \left(\frac{\sqrt{a+b} \text{Cosh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} + \frac{\sqrt{a+b} \text{Sinh}[c] \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}}{\sqrt{a}} \right) \right) \text{Sinh}[c] \right) / \right. \\
 & \left. \left(128 b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]} \right) \right) / \left((a+b)^2 (a + b \text{Sech}[c + dx]^2)^3 \right) + \\
 & \left((8a^3 + 20a^2b + 15ab^2) (a + 2b + a \text{Cosh}[2c + 2dx])^3 \right. \\
 & \left. \text{Sech}[c + dx]^6 \right. \\
 & \left. \left(- \frac{i \text{Cosh}[c] \text{Log}[a + 2b + a \text{Cosh}[2c + 2dx]]}{256 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\text{Cosh}[2c] - \text{Sinh}[2c]}} + \right. \right.
 \end{aligned}$$

$$\frac{\left(\frac{\int \frac{\sqrt{a} \operatorname{Cosh}[c] \operatorname{Log}[a + 2b + a \operatorname{Cosh}[2c + 2dx]] \operatorname{Sinh}[c]}{256 \sqrt{a} b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]}} dx}{(a+b)^2 (a+b \operatorname{Sech}[c+dx]^2)^3} + \left((8a^2 + 20ab + 15b^2) (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \right. \right. \right. \\ \left. \left. \left(\frac{\int \frac{\sqrt{a} \operatorname{Cosh}[c] \operatorname{Log}[a + 2b + a \operatorname{Cosh}[2c + 2dx]]}{256 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]}} dx - \frac{\int \frac{\sqrt{a} \operatorname{Log}[a + 2b + a \operatorname{Cosh}[2c + 2dx]] \operatorname{Sinh}[c]}{256 b^3 \sqrt{a+b} d \sqrt{\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]}} dx}{(a+b)^2 (a+b \operatorname{Sech}[c+dx]^2)^3} + \left((a+2b+a \operatorname{Cosh}[2c+2dx])^2 \operatorname{Sech}[c+dx]^6 \right. \right. \right. \right. \\ \left. \left. \left. (-4a^2 \operatorname{Sinh}[c+dx] - 7ab \operatorname{Sinh}[c+dx]) \right) \right) \right) / (32b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+dx]^2)^3) - \frac{a (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[c+dx]^5 \operatorname{Tanh}[c+dx]}{8b (a+b) d (a+b \operatorname{Sech}[c+dx]^2)^3}$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^2 \operatorname{Tanh}[c + dx]^4 dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Tanh}[c + dx]}{d} - \frac{a^2 \operatorname{Tanh}[c + dx]^3}{3d} + \frac{b (2a + b) \operatorname{Tanh}[c + dx]^5}{5d} - \frac{b^2 \operatorname{Tanh}[c + dx]^7}{7d}$$

Result (type 3, 395 leaves):

$$\frac{1}{13440d} \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^7 (3675 a^2 d x \operatorname{Cosh}[dx] + 3675 a^2 d x \operatorname{Cosh}[2c + dx] + 2205 a^2 d x \operatorname{Cosh}[2c + 3dx] + 2205 a^2 d x \operatorname{Cosh}[4c + 3dx] + 735 a^2 d x \operatorname{Cosh}[4c + 5dx] + 735 a^2 d x \operatorname{Cosh}[6c + 5dx] + 105 a^2 d x \operatorname{Cosh}[6c + 7dx] + 105 a^2 d x \operatorname{Cosh}[8c + 7dx] - 5320 a^2 \operatorname{Sinh}[dx] + 1680 ab \operatorname{Sinh}[dx] + 840 b^2 \operatorname{Sinh}[dx] + 4480 a^2 \operatorname{Sinh}[2c + dx] - 1260 ab \operatorname{Sinh}[2c + dx] + 420 b^2 \operatorname{Sinh}[2c + dx] - 3780 a^2 \operatorname{Sinh}[2c + 3dx] + 924 ab \operatorname{Sinh}[2c + 3dx] - 168 b^2 \operatorname{Sinh}[2c + 3dx] + 2100 a^2 \operatorname{Sinh}[4c + 3dx] - 840 ab \operatorname{Sinh}[4c + 3dx] - 420 b^2 \operatorname{Sinh}[4c + 3dx] - 1540 a^2 \operatorname{Sinh}[4c + 5dx] + 168 ab \operatorname{Sinh}[4c + 5dx] + 84 b^2 \operatorname{Sinh}[4c + 5dx] + 420 a^2 \operatorname{Sinh}[6c + 5dx] - 420 ab \operatorname{Sinh}[6c + 5dx] - 280 a^2 \operatorname{Sinh}[6c + 7dx] + 84 ab \operatorname{Sinh}[6c + 7dx] + 12 b^2 \operatorname{Sinh}[6c + 7dx])$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^2 \operatorname{Tanh}[c + dx]^2 dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Tanh}[c + d x]}{d} + \frac{b (2 a + b) \operatorname{Tanh}[c + d x]^3}{3 d} - \frac{b^2 \operatorname{Tanh}[c + d x]^5}{5 d}$$

Result (type 3, 281 leaves):

$$\frac{1}{480 d} \operatorname{Sech}[c] \operatorname{Sech}[c + d x]^5 \left(150 a^2 d x \operatorname{Cosh}[d x] + 150 a^2 d x \operatorname{Cosh}[2 c + d x] + 75 a^2 d x \operatorname{Cosh}[2 c + 3 d x] + 75 a^2 d x \operatorname{Cosh}[4 c + 3 d x] + 15 a^2 d x \operatorname{Cosh}[4 c + 5 d x] + 15 a^2 d x \operatorname{Cosh}[6 c + 5 d x] - 180 a^2 \operatorname{Sinh}[d x] + 80 a b \operatorname{Sinh}[d x] - 20 b^2 \operatorname{Sinh}[d x] + 120 a^2 \operatorname{Sinh}[2 c + d x] - 120 a b \operatorname{Sinh}[2 c + d x] - 60 b^2 \operatorname{Sinh}[2 c + d x] - 120 a^2 \operatorname{Sinh}[2 c + 3 d x] + 40 a b \operatorname{Sinh}[2 c + 3 d x] + 20 b^2 \operatorname{Sinh}[2 c + 3 d x] + 30 a^2 \operatorname{Sinh}[4 c + 3 d x] - 60 a b \operatorname{Sinh}[4 c + 3 d x] - 30 a^2 \operatorname{Sinh}[4 c + 5 d x] + 20 a b \operatorname{Sinh}[4 c + 5 d x] + 4 b^2 \operatorname{Sinh}[4 c + 5 d x] \right)$$

Problem 116: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x])^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b (2 a + b) \operatorname{Tanh}[c + d x]}{d} - \frac{b^2 \operatorname{Tanh}[c + d x]^3}{3 d}$$

Result (type 3, 106 leaves):

$$\left(4 (b + a \operatorname{Cosh}[c + d x])^2 \operatorname{Sech}[c + d x]^3 \left(3 a^2 d x \operatorname{Cosh}[c + d x]^3 + b^2 \operatorname{Sech}[c] \operatorname{Sinh}[d x] + 2 b (3 a + b) \operatorname{Cosh}[c + d x]^2 \operatorname{Sech}[c] \operatorname{Sinh}[d x] + b^2 \operatorname{Cosh}[c + d x] \operatorname{Tanh}[c] \right) \right) / \left(3 d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \right)$$

Problem 118: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^2 (a + b \operatorname{Sech}[c + d x])^2 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$a^2 x - \frac{(a + b)^2 \operatorname{Coth}[c + d x]}{d} - \frac{b^2 \operatorname{Tanh}[c + d x]}{d}$$

Result (type 3, 82 leaves):

$$\left(4 (b + a \operatorname{Cosh}[c + d x])^2 \operatorname{Sech}[c + d x] \left(a^2 d x \operatorname{Cosh}[c + d x] + \left((a + b)^2 \operatorname{Coth}[c + d x] \operatorname{Csch}[c] - b^2 \operatorname{Sech}[c] \right) \operatorname{Sinh}[d x] \right) \right) / \left(d (a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \right)$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^4 (a + b \operatorname{Sech}[c + d x])^2 dx$$

Optimal (type 3, 46 leaves, 4 steps):

$$a^2 x - \frac{(a^2 - b^2) \operatorname{Coth}[c + d x]}{d} - \frac{(a + b)^2 \operatorname{Coth}[c + d x]^3}{3 d}$$

Result (type 3, 160 leaves):

$$\frac{1}{24 d} \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^3 \\ (9 a^2 d x \operatorname{Cosh}[d x] - 9 a^2 d x \operatorname{Cosh}[2 c + d x] - 3 a^2 d x \operatorname{Cosh}[2 c + 3 d x] + 3 a^2 d x \operatorname{Cosh}[4 c + 3 d x] - \\ 12 a^2 \operatorname{Sinh}[d x] + 12 b^2 \operatorname{Sinh}[d x] - 12 a^2 \operatorname{Sinh}[2 c + d x] - 12 a b \operatorname{Sinh}[2 c + d x] + \\ 8 a^2 \operatorname{Sinh}[2 c + 3 d x] + 4 a b \operatorname{Sinh}[2 c + 3 d x] - 4 b^2 \operatorname{Sinh}[2 c + 3 d x])$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + d x]^6 (a + b \operatorname{Sech}[c + d x]^2)^2 dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Coth}[c + d x]}{d} - \frac{(a^2 - b^2) \operatorname{Coth}[c + d x]^3}{3 d} - \frac{(a + b)^2 \operatorname{Coth}[c + d x]^5}{5 d}$$

Result (type 3, 256 leaves):

$$\frac{1}{480 d} \operatorname{Csch}[c] \operatorname{Csch}[c + d x]^5 (-150 a^2 d x \operatorname{Cosh}[d x] + 150 a^2 d x \operatorname{Cosh}[2 c + d x] + 75 a^2 d x \operatorname{Cosh}[2 c + 3 d x] - \\ 75 a^2 d x \operatorname{Cosh}[4 c + 3 d x] - 15 a^2 d x \operatorname{Cosh}[4 c + 5 d x] + 15 a^2 d x \operatorname{Cosh}[6 c + 5 d x] + \\ 280 a^2 \operatorname{Sinh}[d x] + 120 a b \operatorname{Sinh}[d x] + 20 b^2 \operatorname{Sinh}[d x] + 180 a^2 \operatorname{Sinh}[2 c + d x] - \\ 60 b^2 \operatorname{Sinh}[2 c + d x] - 140 a^2 \operatorname{Sinh}[2 c + 3 d x] + 20 b^2 \operatorname{Sinh}[2 c + 3 d x] - 90 a^2 \operatorname{Sinh}[4 c + 3 d x] - \\ 60 a b \operatorname{Sinh}[4 c + 3 d x] + 46 a^2 \operatorname{Sinh}[4 c + 5 d x] + 12 a b \operatorname{Sinh}[4 c + 5 d x] - 4 b^2 \operatorname{Sinh}[4 c + 5 d x])$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + d x]^2)^3 \operatorname{Tanh}[c + d x]^4 dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$a^3 x - \frac{a^3 \operatorname{Tanh}[c + d x]}{d} - \frac{a^3 \operatorname{Tanh}[c + d x]^3}{3 d} + \\ \frac{b (3 a^2 + 3 a b + b^2) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^2 (3 a + 2 b) \operatorname{Tanh}[c + d x]^7}{7 d} + \frac{b^3 \operatorname{Tanh}[c + d x]^9}{9 d}$$

Result (type 3, 683 leaves):

$$\begin{aligned}
 & \frac{8 a^3 x \operatorname{Cosh}[c+d x]^6 (a+b \operatorname{Sech}[c+d x]^2)^3}{(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \\
 & \frac{8 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3 (27 a^2 \operatorname{Sinh}[c]-10 b^3 \operatorname{Sinh}[c])}{63 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \\
 & \left(\frac{8 \operatorname{Cosh}[c+d x]^2 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3 (63 a^2 b \operatorname{Sinh}[c]-72 a b^2 \operatorname{Sinh}[c]+b^3 \operatorname{Sinh}[c])}{(105 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) + (8 \operatorname{Cosh}[c+d x]^4 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3} \right) / \\
 & \left(\frac{105 a^3 \operatorname{Sinh}[c]-378 a^2 b \operatorname{Sinh}[c]+27 a b^2 \operatorname{Sinh}[c]+4 b^3 \operatorname{Sinh}[c]}{(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) +} \right) / \\
 & \frac{8 b^3 \operatorname{Sech}[c] \operatorname{Sech}[c+d x]^3 (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Sinh}[d x]}{9 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3} + \\
 & \left(\frac{8 \operatorname{Sech}[c] \operatorname{Sech}[c+d x] (a+b \operatorname{Sech}[c+d x]^2)^3 (27 a b^2 \operatorname{Sinh}[d x]-10 b^3 \operatorname{Sinh}[d x])}{(63 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) - (8 \operatorname{Cosh}[c+d x]^5 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3} \right) / \\
 & \left(\frac{420 a^3 \operatorname{Sinh}[d x]-189 a^2 b \operatorname{Sinh}[d x]-54 a b^2 \operatorname{Sinh}[d x]-8 b^3 \operatorname{Sinh}[d x]}{(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) + (8 \operatorname{Cosh}[c+d x] \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3} \right) / \\
 & \left(\frac{63 a^2 b \operatorname{Sinh}[d x]-72 a b^2 \operatorname{Sinh}[d x]+b^3 \operatorname{Sinh}[d x]}{(105 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) + (8 \operatorname{Cosh}[c+d x]^3 \operatorname{Sech}[c] (a+b \operatorname{Sech}[c+d x]^2)^3} \right) / \\
 & \left(\frac{105 a^3 \operatorname{Sinh}[d x]-378 a^2 b \operatorname{Sinh}[d x]+27 a b^2 \operatorname{Sinh}[d x]+4 b^3 \operatorname{Sinh}[d x]}{(315 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3) +} \right) + \frac{8 b^3 \operatorname{Sech}[c+d x]^2 (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Tanh}[c]}{9 d (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3}
 \end{aligned}$$

Problem 126: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Sech}[c+d x]^2)^3 \operatorname{Tanh}[c+d x]^2 dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\begin{aligned}
 & a^3 x - \frac{a^3 \operatorname{Tanh}[c+d x]}{d} + \frac{b (3 a^2 + 3 a b + b^2) \operatorname{Tanh}[c+d x]^3}{3 d} - \\
 & \frac{b^2 (3 a + 2 b) \operatorname{Tanh}[c+d x]^5}{5 d} + \frac{b^3 \operatorname{Tanh}[c+d x]^7}{7 d}
 \end{aligned}$$

Result (type 3, 479 leaves):

$$\frac{1}{13440d} \operatorname{Sech}[c] \operatorname{Sech}[c+dx]^7$$

$$\begin{aligned} & (3675 a^3 d x \operatorname{Cosh}[dx] + 3675 a^3 d x \operatorname{Cosh}[2c+dx] + 2205 a^3 d x \operatorname{Cosh}[2c+3dx] + \\ & 2205 a^3 d x \operatorname{Cosh}[4c+3dx] + 735 a^3 d x \operatorname{Cosh}[4c+5dx] + 735 a^3 d x \operatorname{Cosh}[6c+5dx] + \\ & 105 a^3 d x \operatorname{Cosh}[6c+7dx] + 105 a^3 d x \operatorname{Cosh}[8c+7dx] - 4200 a^3 \operatorname{Sinh}[dx] + \\ & 3360 a^2 b \operatorname{Sinh}[dx] + 840 a b^2 \operatorname{Sinh}[dx] - 560 b^3 \operatorname{Sinh}[dx] + \\ & 3150 a^3 \operatorname{Sinh}[2c+dx] - 3990 a^2 b \operatorname{Sinh}[2c+dx] - 2100 a b^2 \operatorname{Sinh}[2c+dx] - \\ & 1120 b^3 \operatorname{Sinh}[2c+dx] - 3150 a^3 \operatorname{Sinh}[2c+3dx] + 1890 a^2 b \operatorname{Sinh}[2c+3dx] + \\ & 504 a b^2 \operatorname{Sinh}[2c+3dx] + 336 b^3 \operatorname{Sinh}[2c+3dx] + 1260 a^3 \operatorname{Sinh}[4c+3dx] - \\ & 2520 a^2 b \operatorname{Sinh}[4c+3dx] - 1260 a b^2 \operatorname{Sinh}[4c+3dx] - 1260 a^3 \operatorname{Sinh}[4c+5dx] + \\ & 840 a^2 b \operatorname{Sinh}[4c+5dx] + 588 a b^2 \operatorname{Sinh}[4c+5dx] + 112 b^3 \operatorname{Sinh}[4c+5dx] + \\ & 210 a^3 \operatorname{Sinh}[6c+5dx] - 630 a^2 b \operatorname{Sinh}[6c+5dx] - 210 a^3 \operatorname{Sinh}[6c+7dx] + \\ & 210 a^2 b \operatorname{Sinh}[6c+7dx] + 84 a b^2 \operatorname{Sinh}[6c+7dx] + 16 b^3 \operatorname{Sinh}[6c+7dx]) \end{aligned}$$

Problem 128: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx])^3 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a^3 x + \frac{b(3a^2 + 3ab + b^2) \operatorname{Tanh}[c + dx]}{d} - \frac{b^2(3a + 2b) \operatorname{Tanh}[c + dx]^3}{3d} + \frac{b^3 \operatorname{Tanh}[c + dx]^5}{5d}$$

Result (type 3, 268 leaves):

$$\frac{1}{480d} \operatorname{Sech}[c] \operatorname{Sech}[c+dx]^5 (150 a^3 d x \operatorname{Cosh}[dx] + 150 a^3 d x \operatorname{Cosh}[2c+dx] + 75 a^3 d x \operatorname{Cosh}[2c+3dx] +$$

$$75 a^3 d x \operatorname{Cosh}[4c+3dx] + 15 a^3 d x \operatorname{Cosh}[4c+5dx] + 15 a^3 d x \operatorname{Cosh}[6c+5dx] +$$

$$540 a^2 b \operatorname{Sinh}[dx] + 420 a b^2 \operatorname{Sinh}[dx] + 160 b^3 \operatorname{Sinh}[dx] -$$

$$360 a^2 b \operatorname{Sinh}[2c+dx] - 180 a b^2 \operatorname{Sinh}[2c+dx] + 360 a^2 b \operatorname{Sinh}[2c+3dx] +$$

$$300 a b^2 \operatorname{Sinh}[2c+3dx] + 80 b^3 \operatorname{Sinh}[2c+3dx] - 90 a^2 b \operatorname{Sinh}[4c+3dx] +$$

$$90 a^2 b \operatorname{Sinh}[4c+5dx] + 60 a b^2 \operatorname{Sinh}[4c+5dx] + 16 b^3 \operatorname{Sinh}[4c+5dx])$$

Problem 130: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[c + dx]^2 (a + b \operatorname{Sech}[c + dx])^3 dx$$

Optimal (type 3, 61 leaves, 4 steps):

$$a^3 x - \frac{(a+b)^3 \operatorname{Coth}[c+dx]}{d} - \frac{b^2(3a+2b) \operatorname{Tanh}[c+dx]}{d} + \frac{b^3 \operatorname{Tanh}[c+dx]^3}{3d}$$

Result (type 3, 126 leaves):

$$\left(8 (a \operatorname{Cosh}[c + dx] + b \operatorname{Sech}[c + dx])^3 (3 a^3 d x \operatorname{Cosh}[c + dx]^3 - b^3 \operatorname{Sech}[c] \operatorname{Sinh}[dx] +$$

$$\operatorname{Cosh}[c + dx]^2 (3 (a + b)^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c] - b^2 (9 a + 5 b) \operatorname{Sech}[c]) \operatorname{Sinh}[dx] -$$

$$b^3 \operatorname{Cosh}[c + dx] \operatorname{Tanh}[c]) \right) / \left(3 d (a + 2 b + a \operatorname{Cosh}[2(c + dx)])^3 \right)$$

Problem 131: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c + d x]^3 (a + b \text{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$-\frac{(a+b)^3 \text{Csch}[c+dx]^2}{2d} + \frac{b^2(3a+2b) \text{Log}[\text{Cosh}[c+dx]]}{d} + \frac{(a-2b)(a+b)^2 \text{Log}[\text{Sinh}[c+dx]]}{d} - \frac{b^3 \text{Sech}[c+dx]^2}{2d}$$

Result (type 3, 174 leaves):

$$-\frac{1}{2d} \text{Csch}[2(c+dx)]^2 \left(2a^3 + 6a^2b + 6ab^2 + 2(a^3 + 3a^2b + 3ab^2 + 2b^3) \text{Cosh}[2(c+dx)] + 3a^2 \text{Log}[\text{Cosh}[c+dx]] + 2b^3 \text{Log}[\text{Cosh}[c+dx]] + a^3 \text{Log}[\text{Sinh}[c+dx]] - 3a^2 \text{Log}[\text{Sinh}[c+dx]] - 2b^3 \text{Log}[\text{Sinh}[c+dx]] - \text{Cosh}[4(c+dx)] (b^2(3a+2b) \text{Log}[\text{Cosh}[c+dx]] + (a-2b)(a+b)^2 \text{Log}[\text{Sinh}[c+dx]]) \right)$$

Problem 132: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c + d x]^4 (a + b \text{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 60 leaves, 4 steps):

$$a^3 x - \frac{(a-2b)(a+b)^2 \text{Coth}[c+dx]}{d} - \frac{(a+b)^3 \text{Coth}[c+dx]^3}{3d} + \frac{b^3 \text{Tanh}[c+dx]}{d}$$

Result (type 3, 343 leaves):

$$\frac{1}{96d} \text{Csch}[c] \text{Csch}[c+dx]^3 \text{Sech}[c] \text{Sech}[c+dx] \left(6a^3 dx \text{Cosh}[2dx] - 3a^3 dx \text{Cosh}[2(c+2dx)] - 6a^3 dx \text{Cosh}[4c+2dx] + 3a^3 dx \text{Cosh}[6c+4dx] - 18a^2 b \text{Sinh}[2c] - 36a^2 b^2 \text{Sinh}[2c] - 4a^3 \text{Sinh}[2dx] + 6a^2 b \text{Sinh}[2dx] + 24a^2 b^2 \text{Sinh}[2dx] + 32b^3 \text{Sinh}[2dx] - 16a^3 \text{Sinh}[2(c+dx)] - 12a^2 b \text{Sinh}[2(c+dx)] + 24a^2 b^2 \text{Sinh}[2(c+dx)] + 8b^3 \text{Sinh}[2(c+dx)] + 8a^3 \text{Sinh}[4(c+dx)] + 6a^2 b \text{Sinh}[4(c+dx)] - 12a^2 b \text{Sinh}[4(c+dx)] - 4b^3 \text{Sinh}[4(c+dx)] + 8a^3 \text{Sinh}[2(c+2dx)] + 6a^2 b \text{Sinh}[2(c+2dx)] - 12a^2 b \text{Sinh}[2(c+2dx)] - 12a^2 b \text{Sinh}[2(c+2dx)] - 16b^3 \text{Sinh}[2(c+2dx)] - 12a^3 \text{Sinh}[4c+2dx] - 18a^2 b \text{Sinh}[4c+2dx] \right)$$

Problem 134: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c + d x]^6 (a + b \text{Sech}[c + d x]^2)^3 dx$$

Optimal (type 3, 69 leaves, 4 steps):

$$a^3 x - \frac{(a^3 + b^3) \operatorname{Coth}[c + dx]}{d} - \frac{(a - 2b)(a + b)^2 \operatorname{Coth}[c + dx]^3}{3d} - \frac{(a + b)^3 \operatorname{Coth}[c + dx]^5}{5d}$$

Result (type 3, 303 leaves):

$$\frac{1}{480d} \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^5 \left(-150 a^3 dx \operatorname{Cosh}[dx] + 150 a^3 dx \operatorname{Cosh}[2c + dx] + 75 a^3 dx \operatorname{Cosh}[2c + 3dx] - 75 a^3 dx \operatorname{Cosh}[4c + 3dx] - 15 a^3 dx \operatorname{Cosh}[4c + 5dx] + 15 a^3 dx \operatorname{Cosh}[6c + 5dx] + 280 a^3 \operatorname{Sinh}[dx] + 180 a^2 b \operatorname{Sinh}[dx] + 60 a b^2 \operatorname{Sinh}[dx] + 160 b^3 \operatorname{Sinh}[dx] + 180 a^3 \operatorname{Sinh}[2c + dx] - 180 a b^2 \operatorname{Sinh}[2c + dx] - 140 a^3 \operatorname{Sinh}[2c + 3dx] + 60 a b^2 \operatorname{Sinh}[2c + 3dx] - 80 b^3 \operatorname{Sinh}[2c + 3dx] - 90 a^3 \operatorname{Sinh}[4c + 3dx] - 90 a^2 b \operatorname{Sinh}[4c + 3dx] + 46 a^3 \operatorname{Sinh}[4c + 5dx] + 18 a^2 b \operatorname{Sinh}[4c + 5dx] - 12 a b^2 \operatorname{Sinh}[4c + 5dx] + 16 b^3 \operatorname{Sinh}[4c + 5dx] \right)$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^4 dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$a^4 x + \frac{b(2a + b)(2a^2 + 2ab + b^2) \operatorname{Tanh}[c + dx]}{d} - \frac{b^2(6a^2 + 8ab + 3b^2) \operatorname{Tanh}[c + dx]^3}{3d} + \frac{b^3(4a + 3b) \operatorname{Tanh}[c + dx]^5}{5d} - \frac{b^4 \operatorname{Tanh}[c + dx]^7}{7d}$$

Result (type 3, 455 leaves):

$$\frac{1}{13440d} \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^7 \left(3675 a^4 dx \operatorname{Cosh}[dx] + 3675 a^4 dx \operatorname{Cosh}[2c + dx] + 2205 a^4 dx \operatorname{Cosh}[2c + 3dx] + 2205 a^4 dx \operatorname{Cosh}[4c + 3dx] + 735 a^4 dx \operatorname{Cosh}[4c + 5dx] + 735 a^4 dx \operatorname{Cosh}[6c + 5dx] + 105 a^4 dx \operatorname{Cosh}[6c + 7dx] + 105 a^4 dx \operatorname{Cosh}[8c + 7dx] + 16800 a^3 b \operatorname{Sinh}[dx] + 18480 a^2 b^2 \operatorname{Sinh}[dx] + 11200 a b^3 \operatorname{Sinh}[dx] + 3360 b^4 \operatorname{Sinh}[dx] - 12600 a^3 b \operatorname{Sinh}[2c + dx] - 10920 a^2 b^2 \operatorname{Sinh}[2c + dx] - 4480 a b^3 \operatorname{Sinh}[2c + dx] + 12600 a^3 b \operatorname{Sinh}[2c + 3dx] + 15120 a^2 b^2 \operatorname{Sinh}[2c + 3dx] + 9408 a b^3 \operatorname{Sinh}[2c + 3dx] + 2016 b^4 \operatorname{Sinh}[2c + 3dx] - 5040 a^3 b \operatorname{Sinh}[4c + 3dx] - 2520 a^2 b^2 \operatorname{Sinh}[4c + 3dx] + 5040 a^3 b \operatorname{Sinh}[4c + 5dx] + 5880 a^2 b^2 \operatorname{Sinh}[4c + 5dx] + 3136 a b^3 \operatorname{Sinh}[4c + 5dx] + 672 b^4 \operatorname{Sinh}[4c + 5dx] - 840 a^3 b \operatorname{Sinh}[6c + 5dx] + 840 a^3 b \operatorname{Sinh}[6c + 7dx] + 840 a^2 b^2 \operatorname{Sinh}[6c + 7dx] + 448 a b^3 \operatorname{Sinh}[6c + 7dx] + 96 b^4 \operatorname{Sinh}[6c + 7dx] \right)$$

Problem 137: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[c + dx]^2)^5 dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$a^5 x + \frac{b (5 a^4 + 10 a^3 b + 10 a^2 b^2 + 5 a b^3 + b^4) \operatorname{Tanh}[c + d x]}{d} - \frac{b^2 (10 a^3 + 20 a^2 b + 15 a b^2 + 4 b^3) \operatorname{Tanh}[c + d x]^3}{3 d} + \frac{b^3 (10 a^2 + 15 a b + 6 b^2) \operatorname{Tanh}[c + d x]^5}{5 d} - \frac{b^4 (5 a + 4 b) \operatorname{Tanh}[c + d x]^7}{7 d} + \frac{b^5 \operatorname{Tanh}[c + d x]^9}{9 d}$$

Result (type 3, 724 leaves):

$$\frac{32 a^5 x \operatorname{Cosh}[c + d x]^{10} (a + b \operatorname{Sech}[c + d x]^2)^5}{(a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5} + \frac{(32 \operatorname{Cosh}[c + d x]^4 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (45 a b^4 \operatorname{Sinh}[c] + 8 b^5 \operatorname{Sinh}[c]))}{(63 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + (64 \operatorname{Cosh}[c + d x]^6 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (105 a^2 b^3 \operatorname{Sinh}[c] + 45 a b^4 \operatorname{Sinh}[c] + 8 b^5 \operatorname{Sinh}[c]))} \Big/ \frac{(105 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + (64 \operatorname{Cosh}[c + d x]^8 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (525 a^3 b^2 \operatorname{Sinh}[c] + 420 a^2 b^3 \operatorname{Sinh}[c] + 180 a b^4 \operatorname{Sinh}[c] + 32 b^5 \operatorname{Sinh}[c]))}{(315 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + \frac{32 b^5 \operatorname{Cosh}[c + d x] \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 \operatorname{Sinh}[d x]}{9 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5} + \frac{(32 \operatorname{Cosh}[c + d x]^3 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (45 a b^4 \operatorname{Sinh}[d x] + 8 b^5 \operatorname{Sinh}[d x]))}{(63 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + (64 \operatorname{Cosh}[c + d x]^5 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (105 a^2 b^3 \operatorname{Sinh}[d x] + 45 a b^4 \operatorname{Sinh}[d x] + 8 b^5 \operatorname{Sinh}[d x]))} \Big/ \frac{(105 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + (64 \operatorname{Cosh}[c + d x]^7 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (525 a^3 b^2 \operatorname{Sinh}[d x] + 420 a^2 b^3 \operatorname{Sinh}[d x] + 180 a b^4 \operatorname{Sinh}[d x] + 32 b^5 \operatorname{Sinh}[d x]))}{(315 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + \frac{(32 \operatorname{Cosh}[c + d x]^9 \operatorname{Sech}[c] (a + b \operatorname{Sech}[c + d x]^2)^5 (1575 a^4 b \operatorname{Sinh}[d x] + 2100 a^3 b^2 \operatorname{Sinh}[d x] + 1680 a^2 b^3 \operatorname{Sinh}[d x] + 720 a b^4 \operatorname{Sinh}[d x] + 128 b^5 \operatorname{Sinh}[d x]))}{(315 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5) + \frac{32 b^5 \operatorname{Cosh}[c + d x]^2 (a + b \operatorname{Sech}[c + d x]^2)^5 \operatorname{Tanh}[c]}{9 d (a + 2 b + a \operatorname{Cosh}[2 c + 2 d x])^5}}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + d x]^5}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$-\frac{(a + 2 b) \operatorname{Log}[\operatorname{Cosh}[c + d x]]}{b^2 d} + \frac{(a + b)^2 \operatorname{Log}[b + a \operatorname{Cosh}[c + d x]^2]}{2 a b^2 d} - \frac{\operatorname{Sech}[c + d x]^2}{2 b d}$$

Result (type 3, 180 leaves):

$$-\frac{1}{8 a b^2 d (a+b \operatorname{Sech}[c+d x]^2)} (a+2 b+a \operatorname{Cosh}[2(c+d x)]) \\ \left(2 a b+2 a(a+2 b) \operatorname{Log}[\operatorname{Cosh}[c+d x]]-a^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]- \right. \\ \left. 2 a b \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]-b^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+\operatorname{Cosh}[2(c+d x)] \right. \\ \left. \left(2 a(a+2 b) \operatorname{Log}[\operatorname{Cosh}[c+d x]]-(a+b)^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]] \right) \right) \operatorname{Sech}[c+d x]^4$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+d x]^4}{a+b \operatorname{Sech}[c+d x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\frac{x}{a}-\frac{(a+b)^{3 / 2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{a b^{3 / 2} d}+\frac{\operatorname{Tanh}[c+d x]}{b d}$$

Result (type 3, 196 leaves):

$$\left((a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^2 \right. \\ \left((a+b)^2 \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[d x] \left(\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c] \right) \left((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x] \right) \right) \right] / \right. \\ \left. \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right) \left(-\operatorname{Cosh}[2 c]+\operatorname{Sinh}[2 c] \right) + \right. \\ \left. \left. \left. \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \left(b d x+a \operatorname{Sech}[c] \operatorname{Sech}[c+d x] \operatorname{Sinh}[d x] \right) \right) \right) \right) / \\ \left(2 a b \sqrt{a+b} d(a+b \operatorname{Sech}[c+d x]^2) \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right)$$

Problem 141: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+d x]^2}{a+b \operatorname{Sech}[c+d x]^2} dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$\frac{x}{a}-\frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{a \sqrt{b} d}$$

Result (type 3, 174 leaves):

$$\left((a+2 b+a \operatorname{Cosh}[2(c+d x)]) \operatorname{Sech}[c+d x]^2 \left(\sqrt{a+b} d x \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4}+(a+b) \right. \right. \\ \left. \left. \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[d x] \left(\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c] \right) \left((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x] \right) \right) \right] / \right. \right. \\ \left. \left. \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right) \left(-\operatorname{Cosh}[2 c]+\operatorname{Sinh}[2 c] \right) \right) \right) \right) / \\ \left(2 a \sqrt{a+b} d(a+b \operatorname{Sech}[c+d x]^2) \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right)$$

Problem 143: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{x}{a} - \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a \sqrt{a + b} d}$$

Result (type 3, 172 leaves):

$$\begin{aligned} & \left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^2 \left(\sqrt{a + b} d x \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \right. \right. \\ & \quad \left. \left. b \operatorname{ArcTanh}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])) \right] \right) \right) / \\ & \quad \left(2 \sqrt{a + b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (-\operatorname{Cosh}[2 c] + \operatorname{Sinh}[2 c]) \Big) / \\ & \left(2 a \sqrt{a + b} d (a + b \operatorname{Sech}[c + d x]^2) \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) \end{aligned}$$

Problem 145: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]^2}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 62 leaves, 6 steps):

$$\frac{x}{a} - \frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{a (a + b)^{3/2} d} - \frac{\operatorname{Coth}[c + d x]}{(a + b) d}$$

Result (type 3, 193 leaves):

$$\begin{aligned} & \left((a + 2 b + a \operatorname{Cosh}[2 (c + d x)]) \operatorname{Sech}[c + d x]^2 \right. \\ & \quad \left(b^2 \operatorname{ArcTanh}\left[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a + 2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c + d x])) \right] \right) / \\ & \quad \left(2 \sqrt{a + b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (-\operatorname{Cosh}[2 c] + \operatorname{Sinh}[2 c]) + \\ & \quad \left. \sqrt{a + b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} ((a + b) d x + a \operatorname{Csch}[c] \operatorname{Csch}[c + d x] \operatorname{Sinh}[d x]) \right) \Big) / \\ & \left(2 a (a + b)^{3/2} d (a + b \operatorname{Sech}[c + d x]^2) \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) \end{aligned}$$

Problem 147: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]^4}{a + b \operatorname{Sech}[c + d x]^2} dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$\frac{x}{a} - \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{a(a+b)^{5/2}d} - \frac{(a+2b) \operatorname{Coth}[c+dx]}{(a+b)^2d} - \frac{\operatorname{Coth}[c+dx]^3}{3(a+b)d}$$

Result (type 3, 581 leaves):

$$\begin{aligned} & \frac{x(a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[c+dx]^2}{2a(a+b \operatorname{Sech}[c+dx]^2)} - \\ & \frac{(a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Coth}[c] \operatorname{Csch}[c+dx]^2 \operatorname{Sech}[c+dx]^2}{6(a+b)d(a+b \operatorname{Sech}[c+dx]^2)} + \\ & \left((a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[c+dx]^2 \left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\ & \quad \left. \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) \right. \right. \\ & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \\ & \quad \left(2a\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]} \right) - \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\ & \quad \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) \right. \\ & \quad \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \Big) / \\ & \quad \left(2a\sqrt{a+b}d\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]} \right) \Big) / \left((a+b)^2(a+b \operatorname{Sech}[c+dx]^2) \right) + \\ & \frac{((a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx]^3 \operatorname{Sech}[c+dx]^2 \operatorname{Sinh}[dx])}{6(a+b)d(a+b \operatorname{Sech}[c+dx]^2)} + \\ & \frac{(a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx] \operatorname{Sech}[c+dx]^2 (4a \operatorname{Sinh}[dx] + 7b \operatorname{Sinh}[dx])}{6(a+b)^2d(a+b \operatorname{Sech}[c+dx]^2)} \end{aligned}$$

Problem 149: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx]^2)^2} dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{2a^2b^{3/2}d} - \frac{(a+b) \operatorname{Tanh}[c+dx]}{2abd(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 228 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \right. \\ \left. \left(2x (a + 2b + a \operatorname{Cosh}[2(c + dx)]) + \left((a^2 - ab - 2b^2) \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \right. \right. \right. \\ \left. \left. \left(\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c] \right) \right) \right) \left(b\sqrt{a+b} d \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \right. \\ \left. \frac{(a+b) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{bd} \right) \left. \right) \left/ \left(8a^2 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) \right.$$

Problem 151: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + dx]^2}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{x}{a^2} - \frac{(a + 2b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{2a^2 \sqrt{b} \sqrt{a+b} d} - \frac{\operatorname{Tanh}[c + dx]}{2ad (a + b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 326 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)])^2 \operatorname{Sech}[c + dx]^4 \right. \\ \left(\frac{16x}{a^2} - \frac{(a + 2b) \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}} \right]}{b^{3/2} (a + b)^{3/2} d} + \left((a^3 - 6a^2b - 24ab^2 - 16b^3) \right. \right. \\ \left. \left. \operatorname{ArcTanh} \left[\frac{\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])}{2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4}} \right] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \right) \right) \left. \right. \\ \left(a^2 b (a + b)^{3/2} d \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + \\ \frac{(a^2 + 8ab + 8b^2) \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{a^2 b (a + b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} + \\ \left. \frac{a \operatorname{Sinh}[2(c + dx)]}{b(a + b) d (a + 2b + a \operatorname{Cosh}[2(c + dx)])} \right) \left. \right) \left/ \left(64 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) \right.$$

Problem 153: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + d x])^2} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} d} - \frac{b \operatorname{Tanh}[c + d x]}{2a (a+b) d (a+b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 221 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + d x)]) \operatorname{Sech}[c + d x]^4 \right. \\ \left. \left(2x (a + 2b + a \operatorname{Cosh}[2(c + d x)]) - \left(b (3a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}} \right] \right) \right) \right) / \\ \left(\operatorname{Sech}[d x] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2c + d x]) \right) / \\ \left(2\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (a + 2b + a \operatorname{Cosh}[2(c + d x)]) \\ (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \left. \right) / \left((a+b)^{3/2} d \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \right. \\ \left. \frac{b \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2d x])}{(a+b) d} \right) / (8a^2 (a+b \operatorname{Sech}[c + d x]^2)^2)$$

Problem 155: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + d x]^2}{(a + b \operatorname{Sech}[c + d x])^2} dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$\frac{x}{a^2} - \frac{b^{3/2} (5a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{5/2} d} - \\ \frac{(2a - b) \operatorname{Coth}[c + d x]}{2a (a+b)^2 d} - \frac{b \operatorname{Coth}[c + d x]}{2a (a+b) d (a+b - b \operatorname{Tanh}[c + d x]^2)}$$

Result (type 3, 268 leaves):

$$\left((a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Sech}[c + dx]^4 \right. \\
 \left. \left(\frac{2x(a + 2b + a \operatorname{Cosh}[2(c + dx)])}{a^2} - (b^2(5a + 2b) \operatorname{ArcTanh}[\right. \right. \\
 \left. \left. (\operatorname{Sech}[dx] (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) ((a + 2b) \operatorname{Sinh}[dx] - a \operatorname{Sinh}[2c + dx])) \right] \right) / \\
 \left(2\sqrt{a+b} \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (a + 2b + a \operatorname{Cosh}[2(c + dx)]) \\
 (\operatorname{Cosh}[2c] - \operatorname{Sinh}[2c]) \Big/ (a^2(a+b)^{5/2} d \sqrt{b(\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} + \\
 \frac{2(a + 2b + a \operatorname{Cosh}[2(c + dx)]) \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \operatorname{Sinh}[dx]}{(a+b)^2 d} + \\
 \left. \frac{b^2 \operatorname{Sech}[2c] ((a + 2b) \operatorname{Sinh}[2c] - a \operatorname{Sinh}[2dx])}{a^2(a+b)^2 d} \right) \Big/ (8(a+b \operatorname{Sech}[c + dx]^2)^2)$$

Problem 157: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + dx]^4}{(a + b \operatorname{Sech}[c + dx]^2)^2} dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$\frac{x}{a^2} - \frac{b^{5/2}(7a + 2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + dx]}{\sqrt{a+b}}\right]}{2a^2(a+b)^{7/2}d} - \frac{(2a^2 + 6ab - b^2) \operatorname{Coth}[c + dx]}{2a(a+b)^3d} - \\
 \frac{(2a - 3b) \operatorname{Coth}[c + dx]^3}{6a(a+b)^2d} - \frac{b \operatorname{Coth}[c + dx]^3}{2a(a+b)d(a+b - b \operatorname{Tanh}[c + dx]^2)}$$

Result (type 3, 685 leaves):

$$\begin{aligned}
& x \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4}{4a^2 (a + b \operatorname{Sech}[c + dx]^2)^2} - \\
& \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Coth}[c] \operatorname{Csch}[c + dx]^2 \operatorname{Sech}[c + dx]^4}{12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2} + \\
& \left((7a + 2b) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[c + dx]^4 \left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\
& \quad \left. \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
& \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) \Big/ \\
& \quad \left(8a^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
& \quad \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
& \quad \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \Big/ \\
& \quad \left. \left(8a^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) \Big/ \left((a + b)^3 (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + dx]^3 \operatorname{Sech}[c + dx]^4 \right. \\
& \quad \left. \operatorname{Sinh}[dx] \right) \Big/ \\
& \left(12(a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Csch}[c] \operatorname{Csch}[c + dx] \right. \\
& \quad \left. \operatorname{Sech}[c + dx]^4 (2a \operatorname{Sinh}[dx] + 5b \operatorname{Sinh}[dx]) \right) \Big/ \\
& \left(6(a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right) + \\
& \left((a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^4 \right. \\
& \quad \left. (a b^3 \operatorname{Sinh}[2c] + 2b^4 \operatorname{Sinh}[2c] - a b^3 \operatorname{Sinh}[2dx]) \right) \Big/ \\
& \left(8a^2 (a + b)^3 d (a + b \operatorname{Sech}[c + dx]^2)^2 \right)
\end{aligned}$$

Problem 158: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c + dx]^6}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 148 leaves, 7 steps):

$$\frac{x}{a^3} - \frac{\sqrt{a+b} (3a^2 - 4ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8a^3 b^{5/2} d} - \frac{(a+b) \operatorname{Tanh}[c+dx]^3}{4abd(a+b-b \operatorname{Tanh}[c+dx]^2)^2} + \frac{(3a-4b)(a+b) \operatorname{Tanh}[c+dx]}{8a^2 b^2 d(a+b-b \operatorname{Tanh}[c+dx]^2)}$$

Result (type 3, 754 leaves):

$$\frac{1}{(a+b \operatorname{Sech}[c+dx]^2)^3} (3a^3 - a^2b + 4ab^2 + 8b^3) (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / \left(64a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / \left(64a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \right) + \frac{1}{128a^3 b^2 d (a+b \operatorname{Sech}[c+dx]^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c+dx]^6 (24a^2 b^2 dx \operatorname{Cosh}[2c] + 64ab^3 dx \operatorname{Cosh}[2c] + 64b^4 dx \operatorname{Cosh}[2c] + 16a^2 b^2 dx \operatorname{Cosh}[2dx] + 32ab^3 dx \operatorname{Cosh}[2dx] + 16a^2 b^2 dx \operatorname{Cosh}[4c+2dx] + 32ab^3 dx \operatorname{Cosh}[4c+2dx] + 4a^2 b^2 dx \operatorname{Cosh}[2c+4dx] + 4a^2 b^2 dx \operatorname{Cosh}[6c+4dx] - 9a^4 \operatorname{Sinh}[2c] - 15a^3 b \operatorname{Sinh}[2c] + 18a^2 b^2 \operatorname{Sinh}[2c] + 72ab^3 \operatorname{Sinh}[2c] + 48b^4 \operatorname{Sinh}[2c] + 9a^4 \operatorname{Sinh}[2dx] + 13a^3 b \operatorname{Sinh}[2dx] - 28a^2 b^2 \operatorname{Sinh}[2dx] - 32ab^3 \operatorname{Sinh}[2dx] - 3a^4 \operatorname{Sinh}[4c+2dx] + a^3 b \operatorname{Sinh}[4c+2dx] + 20a^2 b^2 \operatorname{Sinh}[4c+2dx] + 16ab^3 \operatorname{Sinh}[4c+2dx] + 3a^4 \operatorname{Sinh}[2c+4dx] - 3a^3 b \operatorname{Sinh}[2c+4dx] - 6a^2 b^2 \operatorname{Sinh}[2c+4dx])$$

Problem 160: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+dx]^4}{(a+b \operatorname{Sech}[c+dx]^2)^3} dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{x}{a^3} + \frac{(a^2 - 4 a b - 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8 a^3 b^{3/2} \sqrt{a+b} d} - \frac{(a+b) \operatorname{Tanh}[c+dx]}{4 a b d (a+b-b \operatorname{Tanh}[c+dx])^2} + \frac{(a-4 b) \operatorname{Tanh}[c+dx]}{8 a^2 b d (a+b-b \operatorname{Tanh}[c+dx])^2}$$

Result (type 3, 1730 leaves):

$$\left((a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(\frac{(3a^2+8ab+8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{(a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)])}{((a+b)^2(a+2b+a \operatorname{Cosh}[2(c+dx)])^2)} \right) \right) / (1024 b^{5/2} d (a+b \operatorname{Sech}[c+dx])^2)^3 - \left((a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(-\frac{3a(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{(\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab+4b^2) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)])}{((a+b)^2(a+2b+a \operatorname{Cosh}[2(c+dx)])^2)} \right) \right) / (2048 b^{5/2} d (a+b \operatorname{Sech}[c+dx])^2)^3 + \frac{1}{32 (a+b \operatorname{Sech}[c+dx])^2)^3} (a+2b+a \operatorname{Cosh}[2c+2dx])^3 \operatorname{Sech}[c+dx]^6 \left(\frac{1}{(a+b)^2} (3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5) \left(\left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) (-a \operatorname{Sinh}[dx]-2b \operatorname{Sinh}[dx]+a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) / (64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}) - \left(\operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b}\sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}} \right) (-a \operatorname{Sinh}[dx]-2b \operatorname{Sinh}[dx]+a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) / (64 a^3 b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c]-b \operatorname{Sinh}[4c]}) \right) + \frac{1}{128 a^3 b^2 (a+b)^2 d (a+2b+a \operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c]$$

$$\begin{aligned}
 & \left(768 a^4 b^2 d x \operatorname{Cosh}[2 c] + 3584 a^3 b^3 d x \operatorname{Cosh}[2 c] + 6912 a^2 b^4 d x \operatorname{Cosh}[2 c] + \right. \\
 & 6144 a b^5 d x \operatorname{Cosh}[2 c] + 2048 b^6 d x \operatorname{Cosh}[2 c] + 512 a^4 b^2 d x \operatorname{Cosh}[2 d x] + \\
 & 2048 a^3 b^3 d x \operatorname{Cosh}[2 d x] + 2560 a^2 b^4 d x \operatorname{Cosh}[2 d x] + 1024 a b^5 d x \operatorname{Cosh}[2 d x] + \\
 & 512 a^4 b^2 d x \operatorname{Cosh}[4 c + 2 d x] + 2048 a^3 b^3 d x \operatorname{Cosh}[4 c + 2 d x] + \\
 & 2560 a^2 b^4 d x \operatorname{Cosh}[4 c + 2 d x] + 1024 a b^5 d x \operatorname{Cosh}[4 c + 2 d x] + \\
 & 128 a^4 b^2 d x \operatorname{Cosh}[2 c + 4 d x] + 256 a^3 b^3 d x \operatorname{Cosh}[2 c + 4 d x] + \\
 & 128 a^2 b^4 d x \operatorname{Cosh}[2 c + 4 d x] + 128 a^4 b^2 d x \operatorname{Cosh}[6 c + 4 d x] + \\
 & 256 a^3 b^3 d x \operatorname{Cosh}[6 c + 4 d x] + 128 a^2 b^4 d x \operatorname{Cosh}[6 c + 4 d x] - 9 a^6 \operatorname{Sinh}[2 c] + \\
 & 12 a^5 b \operatorname{Sinh}[2 c] + 684 a^4 b^2 \operatorname{Sinh}[2 c] + 2880 a^3 b^3 \operatorname{Sinh}[2 c] + 5280 a^2 b^4 \operatorname{Sinh}[2 c] + \\
 & 4608 a b^5 \operatorname{Sinh}[2 c] + 1536 b^6 \operatorname{Sinh}[2 c] + 9 a^6 \operatorname{Sinh}[2 d x] - 14 a^5 b \operatorname{Sinh}[2 d x] - \\
 & 608 a^4 b^2 \operatorname{Sinh}[2 d x] - 2112 a^3 b^3 \operatorname{Sinh}[2 d x] - 2560 a^2 b^4 \operatorname{Sinh}[2 d x] - \\
 & 1024 a b^5 \operatorname{Sinh}[2 d x] - 3 a^6 \operatorname{Sinh}[4 c + 2 d x] + 10 a^5 b \operatorname{Sinh}[4 c + 2 d x] + \\
 & 304 a^4 b^2 \operatorname{Sinh}[4 c + 2 d x] + 1056 a^3 b^3 \operatorname{Sinh}[4 c + 2 d x] + 1280 a^2 b^4 \operatorname{Sinh}[4 c + 2 d x] + \\
 & 512 a b^5 \operatorname{Sinh}[4 c + 2 d x] + 3 a^6 \operatorname{Sinh}[2 c + 4 d x] - 12 a^5 b \operatorname{Sinh}[2 c + 4 d x] - \\
 & \left. 204 a^4 b^2 \operatorname{Sinh}[2 c + 4 d x] - 384 a^3 b^3 \operatorname{Sinh}[2 c + 4 d x] - 192 a^2 b^4 \operatorname{Sinh}[2 c + 4 d x] \right) - \\
 & \frac{1}{2048 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
 & \operatorname{Sech}[c+d x]^6 \\
 & \left(\left(6 a^2 \operatorname{ArcTanh}[(\operatorname{Sech}[d x] (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) ((a+2 b) \operatorname{Sinh}[d x] - a \operatorname{Sinh}[2 c+d x]))] \right) / \right. \\
 & \left. \left(2 \sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) (\operatorname{Cosh}[2 c] - \operatorname{Sinh}[2 c]) \right) / \\
 & \left(\sqrt{a+b} \sqrt{b (\operatorname{Cosh}[c] - \operatorname{Sinh}[c])^4} \right) + (a \operatorname{Sech}[2 c] \\
 & \left((-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \operatorname{Sinh}[2 d x] + a (-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3) \right. \\
 & \left. \operatorname{Sinh}[2(c+2 d x)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \operatorname{Sinh}[4 c+2 d x] \right) + \\
 & \left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \operatorname{Tanh}[2 c] \right) / \left(a^2 \right. \\
 & \left. (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \right)
 \end{aligned}$$

Problem 162: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{x}{a^3} - \frac{(3 a^2 + 12 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^3 \sqrt{b} (a+b)^{3/2} d} - \frac{\operatorname{Tanh}[c+d x]}{4 a d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{(3 a+4 b) \operatorname{Tanh}[c+d x]}{8 a^2 (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 1730 leaves):

$$\begin{aligned}
 & - \left(\left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \right. \right. \right. \\
 & \quad \left. \left. \left(a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \operatorname{Cosh}[2(c+dx)]) \operatorname{Sinh}[2(c+dx)] \right) / \right. \right. \\
 & \quad \left. \left. \left((a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right) \right) \right) / \left(1024 b^{5/2} d (a+b \operatorname{Sech}[c+dx]^2)^3 \right) \right) - \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \left(- \frac{3a(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \right. \right. \\
 & \quad \left. \left(\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \operatorname{Cosh}[2(c+dx)]) \right) \right. \\
 & \quad \left. \left. \operatorname{Sinh}[2(c+dx)] \right) / \left((a+b)^2 (a+2b+a \operatorname{Cosh}[2(c+dx)])^2 \right) \right) \right) / \\
 & \left(2048 b^{5/2} d (a+b \operatorname{Sech}[c+dx]^2)^3 \right) + \frac{1}{32 (a+b \operatorname{Sech}[c+dx]^2)^3} \\
 & (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \\
 & \operatorname{Sech}[c + dx]^6 \\
 & \left(\frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \left(\left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \right. \\
 & \quad \left. \left. \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) \right) / \\
 & \left(64a^3b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
 & \quad \left. \left(- \frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
 & \quad \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) \right) / \\
 & \left(64a^3b^2 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) + \\
 & \frac{1}{128a^3b^2(a+b)^2d(a+2b+a \operatorname{Cosh}[2c+2dx])^2} \operatorname{Sech}[2c] \\
 & (768a^4b^2dx \operatorname{Cosh}[2c] + 3584a^3b^3dx \operatorname{Cosh}[2c] + 6912a^2b^4dx \operatorname{Cosh}[2c] + \\
 & 6144ab^5dx \operatorname{Cosh}[2c] + 2048b^6dx \operatorname{Cosh}[2c] + 512a^4b^2dx \operatorname{Cosh}[2dx] + \\
 & 2048a^3b^3dx \operatorname{Cosh}[2dx] + 2560a^2b^4dx \operatorname{Cosh}[2dx] + 1024ab^5dx \operatorname{Cosh}[2dx] + \\
 & 512a^4b^2dx \operatorname{Cosh}[4c+2dx] + 2048a^3b^3dx \operatorname{Cosh}[4c+2dx] + \\
 & 2560a^2b^4dx \operatorname{Cosh}[4c+2dx] + 1024ab^5dx \operatorname{Cosh}[4c+2dx] + \\
 & 128a^4b^2dx \operatorname{Cosh}[2c+4dx] + 256a^3b^3dx \operatorname{Cosh}[2c+4dx] +
 \end{aligned}$$

$$\begin{aligned}
 & 128 a^2 b^4 d x \operatorname{Cosh}[2 c + 4 d x] + 128 a^4 b^2 d x \operatorname{Cosh}[6 c + 4 d x] + \\
 & 256 a^3 b^3 d x \operatorname{Cosh}[6 c + 4 d x] + 128 a^2 b^4 d x \operatorname{Cosh}[6 c + 4 d x] - 9 a^6 \operatorname{Sinh}[2 c] + \\
 & 12 a^5 b \operatorname{Sinh}[2 c] + 684 a^4 b^2 \operatorname{Sinh}[2 c] + 2880 a^3 b^3 \operatorname{Sinh}[2 c] + 5280 a^2 b^4 \operatorname{Sinh}[2 c] + \\
 & 4608 a b^5 \operatorname{Sinh}[2 c] + 1536 b^6 \operatorname{Sinh}[2 c] + 9 a^6 \operatorname{Sinh}[2 d x] - 14 a^5 b \operatorname{Sinh}[2 d x] - \\
 & 608 a^4 b^2 \operatorname{Sinh}[2 d x] - 2112 a^3 b^3 \operatorname{Sinh}[2 d x] - 2560 a^2 b^4 \operatorname{Sinh}[2 d x] - \\
 & 1024 a b^5 \operatorname{Sinh}[2 d x] - 3 a^6 \operatorname{Sinh}[4 c + 2 d x] + 10 a^5 b \operatorname{Sinh}[4 c + 2 d x] + \\
 & 304 a^4 b^2 \operatorname{Sinh}[4 c + 2 d x] + 1056 a^3 b^3 \operatorname{Sinh}[4 c + 2 d x] + 1280 a^2 b^4 \operatorname{Sinh}[4 c + 2 d x] + \\
 & 512 a b^5 \operatorname{Sinh}[4 c + 2 d x] + 3 a^6 \operatorname{Sinh}[2 c + 4 d x] - 12 a^5 b \operatorname{Sinh}[2 c + 4 d x] - \\
 & 204 a^4 b^2 \operatorname{Sinh}[2 c + 4 d x] - 384 a^3 b^3 \operatorname{Sinh}[2 c + 4 d x] - 192 a^2 b^4 \operatorname{Sinh}[2 c + 4 d x] - \\
 & \left. \right) + \\
 & \frac{1}{2048 b^2 (a+b)^2 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \\
 & \operatorname{Sech}[c+d x]^6 \\
 & \left(\left(6 a^2 \operatorname{ArcTanh}\left[\left(\operatorname{Sech}[d x] \left(\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c] \right) \left((a+2 b) \operatorname{Sinh}[d x]-a \operatorname{Sinh}[2 c+d x] \right) \right] \right) / \right. \right. \\
 & \left. \left(2 \sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right) \left(\operatorname{Cosh}[2 c]-\operatorname{Sinh}[2 c] \right) \right) / \\
 & \left(\sqrt{a+b} \sqrt{b \left(\operatorname{Cosh}[c]-\operatorname{Sinh}[c] \right)^4} \right) + (a \operatorname{Sech}[2 c] \\
 & \left((-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \operatorname{Sinh}[2 d x] + a (-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3) \right. \\
 & \left. \operatorname{Sinh}[2(c+2 d x)] + (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \operatorname{Sinh}[4 c+2 d x] \right) + \\
 & \left. (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \operatorname{Tanh}[2 c] \right) / \left(a^2 \right. \\
 & \left. (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2 \right)
 \end{aligned}$$

Problem 164: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 146 leaves, 6 steps):

$$\frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^3 (a+b)^{5/2} d} - \\
 \frac{b \operatorname{Tanh}[c+d x]}{4 a (a+b) d (a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{b (7 a+4 b) \operatorname{Tanh}[c+d x]}{8 a^2 (a+b)^2 d (a+b-b \operatorname{Tanh}[c+d x]^2)}$$

Result (type 3, 597 leaves):

$$\begin{aligned}
 & x \frac{(a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6}{8a^3 (a + b \operatorname{Sech}[c + dx]^2)^3} + \frac{1}{(a + b)^2 (a + b \operatorname{Sech}[c + dx]^2)^3} \\
 & (15a^2 + 20ab + 8b^2) (a + 2b + a \operatorname{Cosh}[2c + 2dx])^3 \operatorname{Sech}[c + dx]^6 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right. \\
 & \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Cosh}[2c] \right) / \\
 & \left(64a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \\
 & \left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c + dx]) \right) \operatorname{Sinh}[2c] \right) / \\
 & \left(64a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \left. \right) + \\
 & \left((a + 2b + a \operatorname{Cosh}[2c + 2dx])^2 \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^6 \right. \\
 & \left. (9a^2 b \operatorname{Sinh}[2c] + 28a b^2 \operatorname{Sinh}[2c] + 16b^3 \operatorname{Sinh}[2c] - 9a^2 b \operatorname{Sinh}[2dx] - 6a b^2 \operatorname{Sinh}[2dx]) \right) / \\
 & (64a^3 (a + b)^2 d (a + b \operatorname{Sech}[c + dx]^2)^3) + \\
 & (a + 2b + a \operatorname{Cosh}[2c + 2dx]) \operatorname{Sech}[2c] \operatorname{Sech}[c + dx]^6 \\
 & (-a b^2 \operatorname{Sinh}[2c] - 2b^3 \operatorname{Sinh}[2c] + a b^2 \operatorname{Sinh}[2dx]) / \\
 & (16a^3 (a + b) d (a + b \operatorname{Sech}[c + dx]^2)^3)
 \end{aligned}$$

Problem 165: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c + dx]}{(a + b \operatorname{Sech}[c + dx]^2)^3} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{b^3}{4a^3 (a + b) d (b + a \operatorname{Cosh}[c + dx]^2)^2} + \frac{b^2 (3a + 2b)}{2a^3 (a + b)^2 d (b + a \operatorname{Cosh}[c + dx]^2)} + \\
 & \frac{b (3a^2 + 3ab + b^2) \operatorname{Log}[b + a \operatorname{Cosh}[c + dx]^2]}{2a^3 (a + b)^3 d} + \frac{\operatorname{Log}[\operatorname{Sinh}[c + dx]]}{(a + b)^3 d}
 \end{aligned}$$

Result (type 3, 358 leaves):

$$\begin{aligned}
 & \frac{1}{4 a^3 (a+b)^3 d (a+2 b+a \operatorname{Cosh}[2(c+d x)])^2} \\
 & (12 a^3 b^2+40 a^2 b^3+40 a b^4+12 b^5+9 a^4 b \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+ \\
 & 33 a^3 b^2 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+51 a^2 b^3 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+ \\
 & 32 a b^4 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+8 b^5 \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+ \\
 & 6 a^5 \operatorname{Log}[\operatorname{Sinh}[c+d x]]+16 a^4 b \operatorname{Log}[\operatorname{Sinh}[c+d x]]+ \\
 & 16 a^3 b^2 \operatorname{Log}[\operatorname{Sinh}[c+d x]]+a^2 \operatorname{Cosh}[4(c+d x)] \\
 & (b(3 a^2+3 a b+b^2) \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+2 a^3 \operatorname{Log}[\operatorname{Sinh}[c+d x]])+ \\
 & 4 a \operatorname{Cosh}[2(c+d x)](b^2(3 a^2+5 a b+2 b^2)+b(3 a^3+9 a^2 b+7 a b^2+2 b^3) \\
 & \operatorname{Log}[a+2 b+a \operatorname{Cosh}[2(c+d x)]]+2 a^3(a+2 b) \operatorname{Log}[\operatorname{Sinh}[c+d x]])
 \end{aligned}$$

Problem 166: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+d x]^2}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\begin{aligned}
 & \frac{x}{a^3} - \frac{b^{3/2}(35 a^2+28 a b+8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+d x]}{\sqrt{a+b}}\right]}{8 a^3(a+b)^{7/2} d} - \frac{(8 a^2-11 a b-4 b^2) \operatorname{Coth}[c+d x]}{8 a^2(a+b)^3 d} \\
 & \frac{b \operatorname{Coth}[c+d x]}{4 a(a+b) d(a+b-b \operatorname{Tanh}[c+d x]^2)^2} - \frac{b(9 a+4 b) \operatorname{Coth}[c+d x]}{8 a^2(a+b)^2 d(a+b-b \operatorname{Tanh}[c+d x]^2)}
 \end{aligned}$$

Result (type 3, 2083 leaves):

$$\begin{aligned}
 & \frac{1}{(a+b)^3(a+b \operatorname{Sech}[c+d x]^2)^3} \\
 & (35 a^2+28 a b+8 b^2)(a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+d x]^6 \left(\left(i b^2 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \right. \right. \\
 & \left. \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Cosh}[2 c] \right) / \\
 & \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) - \left(i b^2 \operatorname{ArcTan}[\operatorname{Sech}[d x]] \right. \\
 & \left. \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) \right. \\
 & \left. \left. (-a \operatorname{Sinh}[d x]-2 b \operatorname{Sinh}[d x]+a \operatorname{Sinh}[2 c+d x]) \right] \operatorname{Sinh}[2 c] \right) / \\
 & \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) +
 \end{aligned}$$

$$\frac{1}{512 a^3 (a+b)^3 d (a+b \operatorname{Sech}[c+d x]^2)^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x])$$

$\operatorname{Csch}[c]$
 $\operatorname{Csch}[c+d x]$
 $\operatorname{Sech}[2 c]$
 $\operatorname{Sech}[c+d x]^6$
 $(8 a^5 d x \operatorname{Cosh}[d x] + 56 a^4 b d x \operatorname{Cosh}[d x] + 184 a^3 b^2 d x \operatorname{Cosh}[d x] + 296 a^2 b^3 d x \operatorname{Cosh}[d x] +$
 $224 a b^4 d x \operatorname{Cosh}[d x] + 64 b^5 d x \operatorname{Cosh}[d x] - 12 a^5 d x \operatorname{Cosh}[3 d x] -$
 $68 a^4 b d x \operatorname{Cosh}[3 d x] - 132 a^3 b^2 d x \operatorname{Cosh}[3 d x] - 108 a^2 b^3 d x \operatorname{Cosh}[3 d x] -$
 $32 a b^4 d x \operatorname{Cosh}[3 d x] - 8 a^5 d x \operatorname{Cosh}[2 c-d x] - 56 a^4 b d x \operatorname{Cosh}[2 c-d x] -$
 $184 a^3 b^2 d x \operatorname{Cosh}[2 c-d x] - 296 a^2 b^3 d x \operatorname{Cosh}[2 c-d x] - 224 a b^4 d x \operatorname{Cosh}[2 c-d x] -$
 $64 b^5 d x \operatorname{Cosh}[2 c-d x] - 8 a^5 d x \operatorname{Cosh}[2 c+d x] - 56 a^4 b d x \operatorname{Cosh}[2 c+d x] -$
 $184 a^3 b^2 d x \operatorname{Cosh}[2 c+d x] - 296 a^2 b^3 d x \operatorname{Cosh}[2 c+d x] - 224 a b^4 d x \operatorname{Cosh}[2 c+d x] -$
 $64 b^5 d x \operatorname{Cosh}[2 c+d x] + 8 a^5 d x \operatorname{Cosh}[4 c+d x] + 56 a^4 b d x \operatorname{Cosh}[4 c+d x] +$
 $184 a^3 b^2 d x \operatorname{Cosh}[4 c+d x] + 296 a^2 b^3 d x \operatorname{Cosh}[4 c+d x] + 224 a b^4 d x \operatorname{Cosh}[4 c+d x] +$
 $64 b^5 d x \operatorname{Cosh}[4 c+d x] + 12 a^5 d x \operatorname{Cosh}[2 c+3 d x] + 68 a^4 b d x \operatorname{Cosh}[2 c+3 d x] +$
 $132 a^3 b^2 d x \operatorname{Cosh}[2 c+3 d x] + 108 a^2 b^3 d x \operatorname{Cosh}[2 c+3 d x] + 32 a b^4 d x \operatorname{Cosh}[2 c+3 d x] -$
 $12 a^5 d x \operatorname{Cosh}[4 c+3 d x] - 68 a^4 b d x \operatorname{Cosh}[4 c+3 d x] - 132 a^3 b^2 d x \operatorname{Cosh}[4 c+3 d x] -$
 $108 a^2 b^3 d x \operatorname{Cosh}[4 c+3 d x] - 32 a b^4 d x \operatorname{Cosh}[4 c+3 d x] + 12 a^5 d x \operatorname{Cosh}[6 c+3 d x] +$
 $68 a^4 b d x \operatorname{Cosh}[6 c+3 d x] + 132 a^3 b^2 d x \operatorname{Cosh}[6 c+3 d x] + 108 a^2 b^3 d x \operatorname{Cosh}[6 c+3 d x] +$
 $32 a b^4 d x \operatorname{Cosh}[6 c+3 d x] - 4 a^5 d x \operatorname{Cosh}[2 c+5 d x] - 12 a^4 b d x \operatorname{Cosh}[2 c+5 d x] -$
 $12 a^3 b^2 d x \operatorname{Cosh}[2 c+5 d x] - 4 a^2 b^3 d x \operatorname{Cosh}[2 c+5 d x] + 4 a^5 d x \operatorname{Cosh}[4 c+5 d x] +$
 $12 a^4 b d x \operatorname{Cosh}[4 c+5 d x] + 12 a^3 b^2 d x \operatorname{Cosh}[4 c+5 d x] + 4 a^2 b^3 d x \operatorname{Cosh}[4 c+5 d x] -$
 $4 a^5 d x \operatorname{Cosh}[6 c+5 d x] - 12 a^4 b d x \operatorname{Cosh}[6 c+5 d x] - 12 a^3 b^2 d x \operatorname{Cosh}[6 c+5 d x] -$
 $4 a^2 b^3 d x \operatorname{Cosh}[6 c+5 d x] + 4 a^5 d x \operatorname{Cosh}[8 c+5 d x] + 12 a^4 b d x \operatorname{Cosh}[8 c+5 d x] +$
 $12 a^3 b^2 d x \operatorname{Cosh}[8 c+5 d x] + 4 a^2 b^3 d x \operatorname{Cosh}[8 c+5 d x] - 32 a^5 \operatorname{Sinh}[d x] -$
 $64 a^4 b \operatorname{Sinh}[d x] - 30 a^2 b^3 \operatorname{Sinh}[d x] - 120 a b^4 \operatorname{Sinh}[d x] - 48 b^5 \operatorname{Sinh}[d x] +$
 $32 a^5 \operatorname{Sinh}[3 d x] + 64 a^4 b \operatorname{Sinh}[3 d x] + 26 a^3 b^2 \operatorname{Sinh}[3 d x] + 86 a^2 b^3 \operatorname{Sinh}[3 d x] +$
 $32 a b^4 \operatorname{Sinh}[3 d x] - 48 a^5 \operatorname{Sinh}[2 c-d x] - 128 a^4 b \operatorname{Sinh}[2 c-d x] - 128 a^3 b^2 \operatorname{Sinh}[2 c-d x] -$
 $30 a^2 b^3 \operatorname{Sinh}[2 c-d x] - 120 a b^4 \operatorname{Sinh}[2 c-d x] - 48 b^5 \operatorname{Sinh}[2 c-d x] +$
 $48 a^5 \operatorname{Sinh}[2 c+d x] + 128 a^4 b \operatorname{Sinh}[2 c+d x] + 102 a^3 b^2 \operatorname{Sinh}[2 c+d x] -$
 $86 a^2 b^3 \operatorname{Sinh}[2 c+d x] - 136 a b^4 \operatorname{Sinh}[2 c+d x] - 48 b^5 \operatorname{Sinh}[2 c+d x] - 32 a^5 \operatorname{Sinh}[4 c+d x] -$
 $64 a^4 b \operatorname{Sinh}[4 c+d x] + 26 a^3 b^2 \operatorname{Sinh}[4 c+d x] + 86 a^2 b^3 \operatorname{Sinh}[4 c+d x] +$
 $136 a b^4 \operatorname{Sinh}[4 c+d x] + 48 b^5 \operatorname{Sinh}[4 c+d x] - 8 a^5 \operatorname{Sinh}[2 c+3 d x] -$
 $26 a^3 b^2 \operatorname{Sinh}[2 c+3 d x] - 86 a^2 b^3 \operatorname{Sinh}[2 c+3 d x] - 32 a b^4 \operatorname{Sinh}[2 c+3 d x] +$
 $32 a^5 \operatorname{Sinh}[4 c+3 d x] + 64 a^4 b \operatorname{Sinh}[4 c+3 d x] - 13 a^3 b^2 \operatorname{Sinh}[4 c+3 d x] -$
 $36 a^2 b^3 \operatorname{Sinh}[4 c+3 d x] - 16 a b^4 \operatorname{Sinh}[4 c+3 d x] - 8 a^5 \operatorname{Sinh}[6 c+3 d x] +$
 $13 a^3 b^2 \operatorname{Sinh}[6 c+3 d x] + 36 a^2 b^3 \operatorname{Sinh}[6 c+3 d x] + 16 a b^4 \operatorname{Sinh}[6 c+3 d x] +$
 $8 a^5 \operatorname{Sinh}[2 c+5 d x] + 13 a^3 b^2 \operatorname{Sinh}[2 c+5 d x] + 6 a^2 b^3 \operatorname{Sinh}[2 c+5 d x] -$
 $13 a^3 b^2 \operatorname{Sinh}[4 c+5 d x] - 6 a^2 b^3 \operatorname{Sinh}[4 c+5 d x] + 8 a^5 \operatorname{Sinh}[6 c+5 d x])$

Problem 168: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[c+d x]^4}{(a+b \operatorname{Sech}[c+d x]^2)^3} dx$$

Optimal (type 3, 232 leaves, 9 steps):

$$\frac{x}{a^3} - \frac{b^{5/2} (63 a^2 + 36 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c+dx]}{\sqrt{a+b}}\right]}{8 a^3 (a+b)^{9/2} d} - \frac{(8 a^3 + 32 a^2 b - 15 a b^2 - 4 b^3) \operatorname{Coth}[c+dx]}{8 a^2 (a+b)^4 d} - \frac{(8 a^2 - 39 a b - 12 b^2) \operatorname{Coth}[c+dx]^3}{24 a^2 (a+b)^3 d} - \frac{b \operatorname{Coth}[c+dx]^3}{4 a (a+b) d (a+b-b \operatorname{Tanh}[c+dx])^2} - \frac{b (11 a+4 b) \operatorname{Coth}[c+dx]^3}{8 a^2 (a+b)^2 d (a+b-b \operatorname{Tanh}[c+dx])^2}$$

Result (type 3, 3334 leaves):

$$\frac{1}{(a+b)^4 (a+b \operatorname{Sech}[c+dx])^3} (63 a^2 + 36 a b + 8 b^2) (a+2 b+a \operatorname{Cosh}[2 c+2 d x])^3 \operatorname{Sech}[c+dx]^6 \left(\left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[dx] - 2 b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2 c+dx]) \right) \operatorname{Cosh}[2 c] \right) / \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) - \left(i b^3 \operatorname{ArcTan}[\operatorname{Sech}[dx]] \left(-\frac{i \operatorname{Cosh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} + \frac{i \operatorname{Sinh}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]}} \right) (-a \operatorname{Sinh}[dx] - 2 b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2 c+dx]) \right) \operatorname{Sinh}[2 c] \right) / \left(64 a^3 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4 c]-b \operatorname{Sinh}[4 c]} \right) \right) + \frac{1}{6144 a^3 (a+b)^4 d (a+b \operatorname{Sech}[c+dx])^3} (a+2 b+a \operatorname{Cosh}[2 c+2 d x]) \operatorname{Csch}[c] \operatorname{Csch}[c+dx]^3 \operatorname{Sech}[2 c] \operatorname{Sech}[c+dx]^6 (-36 a^6 d x \operatorname{Cosh}[dx] - 336 a^5 b d x \operatorname{Cosh}[dx] - 1560 a^4 b^2 d x \operatorname{Cosh}[dx] - 3600 a^3 b^3 d x \operatorname{Cosh}[dx] - 4260 a^2 b^4 d x \operatorname{Cosh}[dx] - 2496 a b^5 d x \operatorname{Cosh}[dx] - 576 b^6 d x \operatorname{Cosh}[dx] + 36 a^6 d x \operatorname{Cosh}[3 d x] + 240 a^5 b d x \operatorname{Cosh}[3 d x] + 408 a^4 b^2 d x \operatorname{Cosh}[3 d x] - 48 a^3 b^3 d x \operatorname{Cosh}[3 d x] - 732 a^2 b^4 d x \operatorname{Cosh}[3 d x] - 672 a b^5 d x \operatorname{Cosh}[3 d x] - 192 b^6 d x \operatorname{Cosh}[3 d x] + 36 a^6 d x \operatorname{Cosh}[2 c-d x] + 336 a^5 b d x \operatorname{Cosh}[2 c-d x] + 1560 a^4 b^2 d x \operatorname{Cosh}[2 c-d x] + 3600 a^3 b^3 d x \operatorname{Cosh}[2 c-d x] + 4260 a^2 b^4 d x \operatorname{Cosh}[2 c-d x] + 2496 a b^5 d x \operatorname{Cosh}[2 c-d x] + 576 b^6 d x \operatorname{Cosh}[2 c-d x] + 36 a^6 d x \operatorname{Cosh}[2 c+dx] + 336 a^5 b d x \operatorname{Cosh}[2 c+dx] + 1560 a^4 b^2 d x \operatorname{Cosh}[2 c+dx] + 3600 a^3 b^3 d x \operatorname{Cosh}[2 c+dx] + 4260 a^2 b^4 d x \operatorname{Cosh}[2 c+dx] + 2496 a b^5 d x \operatorname{Cosh}[2 c+dx] + 576 b^6 d x \operatorname{Cosh}[2 c+dx] - 36 a^6 d x \operatorname{Cosh}[4 c+dx] - 336 a^5 b d x \operatorname{Cosh}[4 c+dx] - 1560 a^4 b^2 d x \operatorname{Cosh}[4 c+dx] - 3600 a^3 b^3 d x \operatorname{Cosh}[4 c+dx] - 4260 a^2 b^4 d x \operatorname{Cosh}[4 c+dx] - 2496 a b^5 d x \operatorname{Cosh}[4 c+dx] - 576 b^6 d x \operatorname{Cosh}[4 c+dx] - 36 a^6 d x \operatorname{Cosh}[2 c+3 d x] -$$

$$\begin{aligned}
& 240 a^5 b d x \operatorname{Cosh}[2 c+3 d x]-408 a^4 b^2 d x \operatorname{Cosh}[2 c+3 d x]+48 a^3 b^3 d x \operatorname{Cosh}[2 c+3 d x]+ \\
& 732 a^2 b^4 d x \operatorname{Cosh}[2 c+3 d x]+672 a b^5 d x \operatorname{Cosh}[2 c+3 d x]+192 b^6 d x \operatorname{Cosh}[2 c+3 d x]+ \\
& 36 a^6 d x \operatorname{Cosh}[4 c+3 d x]+240 a^5 b d x \operatorname{Cosh}[4 c+3 d x]+408 a^4 b^2 d x \operatorname{Cosh}[4 c+3 d x]- \\
& 48 a^3 b^3 d x \operatorname{Cosh}[4 c+3 d x]-732 a^2 b^4 d x \operatorname{Cosh}[4 c+3 d x]-672 a b^5 d x \operatorname{Cosh}[4 c+3 d x]- \\
& 192 b^6 d x \operatorname{Cosh}[4 c+3 d x]-36 a^6 d x \operatorname{Cosh}[6 c+3 d x]-240 a^5 b d x \operatorname{Cosh}[6 c+3 d x]- \\
& 408 a^4 b^2 d x \operatorname{Cosh}[6 c+3 d x]+48 a^3 b^3 d x \operatorname{Cosh}[6 c+3 d x]+732 a^2 b^4 d x \operatorname{Cosh}[6 c+3 d x]+ \\
& 672 a b^5 d x \operatorname{Cosh}[6 c+3 d x]+192 b^6 d x \operatorname{Cosh}[6 c+3 d x]-12 a^6 d x \operatorname{Cosh}[2 c+5 d x]- \\
& 144 a^5 b d x \operatorname{Cosh}[2 c+5 d x]-456 a^4 b^2 d x \operatorname{Cosh}[2 c+5 d x]-624 a^3 b^3 d x \operatorname{Cosh}[2 c+5 d x]- \\
& 396 a^2 b^4 d x \operatorname{Cosh}[2 c+5 d x]-96 a b^5 d x \operatorname{Cosh}[2 c+5 d x]+12 a^6 d x \operatorname{Cosh}[4 c+5 d x]+ \\
& 144 a^5 b d x \operatorname{Cosh}[4 c+5 d x]+456 a^4 b^2 d x \operatorname{Cosh}[4 c+5 d x]+624 a^3 b^3 d x \operatorname{Cosh}[4 c+5 d x]+ \\
& 396 a^2 b^4 d x \operatorname{Cosh}[4 c+5 d x]+96 a b^5 d x \operatorname{Cosh}[4 c+5 d x]-12 a^6 d x \operatorname{Cosh}[6 c+5 d x]- \\
& 144 a^5 b d x \operatorname{Cosh}[6 c+5 d x]-456 a^4 b^2 d x \operatorname{Cosh}[6 c+5 d x]-624 a^3 b^3 d x \operatorname{Cosh}[6 c+5 d x]- \\
& 396 a^2 b^4 d x \operatorname{Cosh}[6 c+5 d x]-96 a b^5 d x \operatorname{Cosh}[6 c+5 d x]+12 a^6 d x \operatorname{Cosh}[8 c+5 d x]+ \\
& 144 a^5 b d x \operatorname{Cosh}[8 c+5 d x]+456 a^4 b^2 d x \operatorname{Cosh}[8 c+5 d x]+624 a^3 b^3 d x \operatorname{Cosh}[8 c+5 d x]+ \\
& 396 a^2 b^4 d x \operatorname{Cosh}[8 c+5 d x]+96 a b^5 d x \operatorname{Cosh}[8 c+5 d x]-12 a^6 d x \operatorname{Cosh}[4 c+7 d x]- \\
& 48 a^5 b d x \operatorname{Cosh}[4 c+7 d x]-72 a^4 b^2 d x \operatorname{Cosh}[4 c+7 d x]-48 a^3 b^3 d x \operatorname{Cosh}[4 c+7 d x]- \\
& 12 a^2 b^4 d x \operatorname{Cosh}[4 c+7 d x]+12 a^6 d x \operatorname{Cosh}[6 c+7 d x]+48 a^5 b d x \operatorname{Cosh}[6 c+7 d x]+ \\
& 72 a^4 b^2 d x \operatorname{Cosh}[6 c+7 d x]+48 a^3 b^3 d x \operatorname{Cosh}[6 c+7 d x]+12 a^2 b^4 d x \operatorname{Cosh}[6 c+7 d x]- \\
& 12 a^6 d x \operatorname{Cosh}[8 c+7 d x]-48 a^5 b d x \operatorname{Cosh}[8 c+7 d x]-72 a^4 b^2 d x \operatorname{Cosh}[8 c+7 d x]- \\
& 48 a^3 b^3 d x \operatorname{Cosh}[8 c+7 d x]-12 a^2 b^4 d x \operatorname{Cosh}[8 c+7 d x]+12 a^6 d x \operatorname{Cosh}[10 c+7 d x]+ \\
& 48 a^5 b d x \operatorname{Cosh}[10 c+7 d x]+72 a^4 b^2 d x \operatorname{Cosh}[10 c+7 d x]+48 a^3 b^3 d x \operatorname{Cosh}[10 c+7 d x]+ \\
& 12 a^2 b^4 d x \operatorname{Cosh}[10 c+7 d x]-128 a^6 \operatorname{Sinh}[d x]-440 a^5 b \operatorname{Sinh}[d x]- \\
& 1152 a^4 b^2 \operatorname{Sinh}[d x]-1920 a^3 b^3 \operatorname{Sinh}[d x]+228 a^2 b^4 \operatorname{Sinh}[d x]+1320 a b^5 \operatorname{Sinh}[d x]+ \\
& 432 b^6 \operatorname{Sinh}[d x]+48 a^6 \operatorname{Sinh}[3 d x]+104 a^5 b \operatorname{Sinh}[3 d x]+640 a^4 b^2 \operatorname{Sinh}[3 d x]+ \\
& 1511 a^3 b^3 \operatorname{Sinh}[3 d x]-528 a^2 b^4 \operatorname{Sinh}[3 d x]+264 a b^5 \operatorname{Sinh}[3 d x]+144 b^6 \operatorname{Sinh}[3 d x]- \\
& 32 a^6 \operatorname{Sinh}[2 c-d x]+384 a^5 b \operatorname{Sinh}[2 c-d x]+2048 a^4 b^2 \operatorname{Sinh}[2 c-d x]+ \\
& 3072 a^3 b^3 \operatorname{Sinh}[2 c-d x]+228 a^2 b^4 \operatorname{Sinh}[2 c-d x]+1320 a b^5 \operatorname{Sinh}[2 c-d x]+ \\
& 432 b^6 \operatorname{Sinh}[2 c-d x]+32 a^6 \operatorname{Sinh}[2 c+d x]-384 a^5 b \operatorname{Sinh}[2 c+d x]- \\
& 2048 a^4 b^2 \operatorname{Sinh}[2 c+d x]-2919 a^3 b^3 \operatorname{Sinh}[2 c+d x]+642 a^2 b^4 \operatorname{Sinh}[2 c+d x]+ \\
& 1416 a b^5 \operatorname{Sinh}[2 c+d x]+432 b^6 \operatorname{Sinh}[2 c+d x]-128 a^6 \operatorname{Sinh}[4 c+d x]- \\
& 440 a^5 b \operatorname{Sinh}[4 c+d x]-1152 a^4 b^2 \operatorname{Sinh}[4 c+d x]-2073 a^3 b^3 \operatorname{Sinh}[4 c+d x]- \\
& 642 a^2 b^4 \operatorname{Sinh}[4 c+d x]-1416 a b^5 \operatorname{Sinh}[4 c+d x]-432 b^6 \operatorname{Sinh}[4 c+d x]- \\
& 144 a^6 \operatorname{Sinh}[2 c+3 d x]-672 a^5 b \operatorname{Sinh}[2 c+3 d x]-960 a^4 b^2 \operatorname{Sinh}[2 c+3 d x]+ \\
& 153 a^3 b^3 \operatorname{Sinh}[2 c+3 d x]+528 a^2 b^4 \operatorname{Sinh}[2 c+3 d x]-264 a b^5 \operatorname{Sinh}[2 c+3 d x]- \\
& 144 b^6 \operatorname{Sinh}[2 c+3 d x]+48 a^6 \operatorname{Sinh}[4 c+3 d x]+104 a^5 b \operatorname{Sinh}[4 c+3 d x]+ \\
& 640 a^4 b^2 \operatorname{Sinh}[4 c+3 d x]+1664 a^3 b^3 \operatorname{Sinh}[4 c+3 d x]-66 a^2 b^4 \operatorname{Sinh}[4 c+3 d x]- \\
& 408 a b^5 \operatorname{Sinh}[4 c+3 d x]-144 b^6 \operatorname{Sinh}[4 c+3 d x]-144 a^6 \operatorname{Sinh}[6 c+3 d x]- \\
& 672 a^5 b \operatorname{Sinh}[6 c+3 d x]-960 a^4 b^2 \operatorname{Sinh}[6 c+3 d x]+66 a^2 b^4 \operatorname{Sinh}[6 c+3 d x]+ \\
& 408 a b^5 \operatorname{Sinh}[6 c+3 d x]+144 b^6 \operatorname{Sinh}[6 c+3 d x]+80 a^6 \operatorname{Sinh}[2 c+5 d x]+ \\
& 480 a^5 b \operatorname{Sinh}[2 c+5 d x]+832 a^4 b^2 \operatorname{Sinh}[2 c+5 d x]+294 a^2 b^4 \operatorname{Sinh}[2 c+5 d x]+ \\
& 96 a b^5 \operatorname{Sinh}[2 c+5 d x]-48 a^6 \operatorname{Sinh}[4 c+5 d x]-120 a^5 b \operatorname{Sinh}[4 c+5 d x]- \\
& 294 a^2 b^4 \operatorname{Sinh}[4 c+5 d x]-96 a b^5 \operatorname{Sinh}[4 c+5 d x]+80 a^6 \operatorname{Sinh}[6 c+5 d x]+ \\
& 480 a^5 b \operatorname{Sinh}[6 c+5 d x]+832 a^4 b^2 \operatorname{Sinh}[6 c+5 d x]-51 a^3 b^3 \operatorname{Sinh}[6 c+5 d x]- \\
& 132 a^2 b^4 \operatorname{Sinh}[6 c+5 d x]-48 a b^5 \operatorname{Sinh}[6 c+5 d x]-48 a^6 \operatorname{Sinh}[8 c+5 d x]- \\
& 120 a^5 b \operatorname{Sinh}[8 c+5 d x]+51 a^3 b^3 \operatorname{Sinh}[8 c+5 d x]+132 a^2 b^4 \operatorname{Sinh}[8 c+5 d x]+ \\
& 48 a b^5 \operatorname{Sinh}[8 c+5 d x]+32 a^6 \operatorname{Sinh}[4 c+7 d x]+104 a^5 b \operatorname{Sinh}[4 c+7 d x]+ \\
& 51 a^3 b^3 \operatorname{Sinh}[4 c+7 d x]+18 a^2 b^4 \operatorname{Sinh}[4 c+7 d x]-51 a^3 b^3 \operatorname{Sinh}[6 c+7 d x]- \\
& 18 a^2 b^4 \operatorname{Sinh}[6 c+7 d x]+32 a^6 \operatorname{Sinh}[8 c+7 d x]+104 a^5 b \operatorname{Sinh}[8 c+7 d x]
\end{aligned}$$

Problem 169: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sech}[c + d x])^4} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$\frac{x}{a^4} - \frac{\sqrt{b} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d x]}{\sqrt{a + b}}\right]}{16 a^4 (a + b)^{7/2} d} - \frac{b \operatorname{Tanh}[c + d x]}{6 a (a + b) d (a + b - b \operatorname{Tanh}[c + d x])^2} - \frac{b (11 a + 6 b) \operatorname{Tanh}[c + d x]}{24 a^2 (a + b)^2 d (a + b - b \operatorname{Tanh}[c + d x])^2} - \frac{b (19 a^2 + 22 a b + 8 b^2) \operatorname{Tanh}[c + d x]}{16 a^3 (a + b)^3 d (a + b - b \operatorname{Tanh}[c + d x])^2}$$

Result (type 3, 1405 leaves):

$$\frac{1}{(a+b)^3 (a+b \operatorname{Sech}[c+dx]^2)^4} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3)$$

$$(a+2b+a \operatorname{Cosh}[2c+2dx])^4 \operatorname{Sech}[c+dx]^8 \left(\left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right. \right.$$

$$\left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Cosh}[2c] \right) /$$

$$\left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) - \left(i b \operatorname{ArcTan}[\operatorname{Sech}[dx]] \right.$$

$$\left. \left(-\frac{i \operatorname{Cosh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} + \frac{i \operatorname{Sinh}[2c]}{2\sqrt{a+b} \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sinh}[dx] - 2b \operatorname{Sinh}[dx] + a \operatorname{Sinh}[2c+dx]) \right) \operatorname{Sinh}[2c] \right) /$$

$$\left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cosh}[4c] - b \operatorname{Sinh}[4c]} \right) \Bigg) +$$

$$\frac{1}{3072 a^4 (a+b)^3 d (a+b \operatorname{Sech}[c+dx]^2)^4}$$

$$(a+2b+a \operatorname{Cosh}[2c+2dx])$$

$$\operatorname{Sech}[2c]$$

$$\operatorname{Sech}[c+dx]^8$$

$$(480 a^6 d x \operatorname{Cosh}[2c] + 3168 a^5 b d x \operatorname{Cosh}[2c] + 8928 a^4 b^2 d x \operatorname{Cosh}[2c] +$$

$$14112 a^3 b^3 d x \operatorname{Cosh}[2c] + 13248 a^2 b^4 d x \operatorname{Cosh}[2c] + 6912 a b^5 d x \operatorname{Cosh}[2c] +$$

$$1536 b^6 d x \operatorname{Cosh}[2c] + 360 a^6 d x \operatorname{Cosh}[2dx] + 2232 a^5 b d x \operatorname{Cosh}[2dx] +$$

$$5688 a^4 b^2 d x \operatorname{Cosh}[2dx] + 7272 a^3 b^3 d x \operatorname{Cosh}[2dx] + 4608 a^2 b^4 d x \operatorname{Cosh}[2dx] +$$

$$1152 a b^5 d x \operatorname{Cosh}[2dx] + 360 a^6 d x \operatorname{Cosh}[4c+2dx] + 2232 a^5 b d x \operatorname{Cosh}[4c+2dx] +$$

$$5688 a^4 b^2 d x \operatorname{Cosh}[4c+2dx] + 7272 a^3 b^3 d x \operatorname{Cosh}[4c+2dx] +$$

$$4608 a^2 b^4 d x \operatorname{Cosh}[4c+2dx] + 1152 a b^5 d x \operatorname{Cosh}[4c+2dx] +$$

$$144 a^6 d x \operatorname{Cosh}[2c+4dx] + 720 a^5 b d x \operatorname{Cosh}[2c+4dx] + 1296 a^4 b^2 d x \operatorname{Cosh}[2c+4dx] +$$

$$1008 a^3 b^3 d x \operatorname{Cosh}[2c+4dx] + 288 a^2 b^4 d x \operatorname{Cosh}[2c+4dx] +$$

$$144 a^6 d x \operatorname{Cosh}[6c+4dx] + 720 a^5 b d x \operatorname{Cosh}[6c+4dx] + 1296 a^4 b^2 d x \operatorname{Cosh}[6c+4dx] +$$

$$1008 a^3 b^3 d x \operatorname{Cosh}[6c+4dx] + 288 a^2 b^4 d x \operatorname{Cosh}[6c+4dx] + 24 a^6 d x \operatorname{Cosh}[4c+6dx] +$$

$$72 a^5 b d x \operatorname{Cosh}[4c+6dx] + 72 a^4 b^2 d x \operatorname{Cosh}[4c+6dx] + 24 a^3 b^3 d x \operatorname{Cosh}[4c+6dx] +$$

$$24 a^6 d x \operatorname{Cosh}[8c+6dx] + 72 a^5 b d x \operatorname{Cosh}[8c+6dx] + 72 a^4 b^2 d x \operatorname{Cosh}[8c+6dx] +$$

$$24 a^3 b^3 d x \operatorname{Cosh}[8c+6dx] + 870 a^5 b \operatorname{Sinh}[2c] + 4292 a^4 b^2 \operatorname{Sinh}[2c] +$$

$$8792 a^3 b^3 \operatorname{Sinh}[2c] + 9936 a^2 b^4 \operatorname{Sinh}[2c] + 5824 a b^5 \operatorname{Sinh}[2c] + 1408 b^6 \operatorname{Sinh}[2c] -$$

$$870 a^5 b \operatorname{Sinh}[2dx] - 3792 a^4 b^2 \operatorname{Sinh}[2dx] - 6432 a^3 b^3 \operatorname{Sinh}[2dx] -$$

$$4608 a^2 b^4 \operatorname{Sinh}[2dx] - 1248 a b^5 \operatorname{Sinh}[2dx] + 435 a^5 b \operatorname{Sinh}[4c+2dx] +$$

$$2124 a^4 b^2 \operatorname{Sinh}[4c+2dx] + 3972 a^3 b^3 \operatorname{Sinh}[4c+2dx] + 3072 a^2 b^4 \operatorname{Sinh}[4c+2dx] +$$

$$864 a b^5 \operatorname{Sinh}[4c+2dx] - 435 a^5 b \operatorname{Sinh}[2c+4dx] - 1374 a^4 b^2 \operatorname{Sinh}[2c+4dx] -$$

$$1248 a^3 b^3 \operatorname{Sinh}[2c+4dx] - 384 a^2 b^4 \operatorname{Sinh}[2c+4dx] + 87 a^5 b \operatorname{Sinh}[6c+4dx] +$$

$$366 a^4 b^2 \operatorname{Sinh}[6c+4dx] + 408 a^3 b^3 \operatorname{Sinh}[6c+4dx] + 144 a^2 b^4 \operatorname{Sinh}[6c+4dx] -$$

$$87 a^5 b \operatorname{Sinh}[4c+6dx] - 116 a^4 b^2 \operatorname{Sinh}[4c+6dx] - 44 a^3 b^3 \operatorname{Sinh}[4c+6dx])$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \operatorname{Sech}[x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[x]}{\sqrt{a + b - b \operatorname{Tanh}[x]^2}}\right] + \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[x]}{\sqrt{a + b - b \operatorname{Tanh}[x]^2}}\right]$$

Result (type 3, 134 leaves):

$$\left(\sqrt{2} \operatorname{Cosh}[x] \left(\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{b} \operatorname{Sinh}[x]}{\sqrt{a + 2b + a \operatorname{Cosh}[2x]}}\right] \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + \sqrt{a} \sqrt{a + b} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Sinh}[x]}{\sqrt{a + b}}\right] \sqrt{\frac{a + 2b + a \operatorname{Cosh}[2x]}{a + b}} \sqrt{a + b \operatorname{Sech}[x]^2} \right) \right) / (a + 2b + a \operatorname{Cosh}[2x])$$

Problem 189: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Sech}[x]^2)^{3/2} \operatorname{Tanh}[x] dx$$

Optimal (type 3, 57 leaves, 6 steps):

$$a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}}\right] - a \sqrt{a + b \operatorname{Sech}[x]^2} - \frac{1}{3} (a + b \operatorname{Sech}[x]^2)^{3/2}$$

Result (type 3, 117 leaves):

$$- \left(\left(2 \left(b \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + 4 a \operatorname{Cosh}[x]^2 \sqrt{a + 2b + a \operatorname{Cosh}[2x]} - 3 \sqrt{2} a^{3/2} \operatorname{Cosh}[x]^3 \operatorname{Log}\left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]}\right] \right) \right) / \left(3 (a + b \operatorname{Sech}[x]^2)^{3/2} \right) \right)$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Coth}[x] (a + b \operatorname{Sech}[x]^2)^{3/2} dx$$

Optimal (type 3, 70 leaves, 8 steps):

$$a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}}\right] - (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a + b}}\right] + b \sqrt{a + b \operatorname{Sech}[x]^2}$$

Result (type 3, 159 leaves):

$$- \left(\left(2 (b + a \operatorname{Cosh}[x])^2 \right. \right. \\ \left. \left. \left(\sqrt{2} (a + b)^2 \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a + b} \operatorname{Cosh}[x]}{\sqrt{a + 2b + a \operatorname{Cosh}[2x]}} \right] \operatorname{Cosh}[x] - \sqrt{a + b} \left(b \sqrt{a + 2b + a \operatorname{Cosh}[2x]} + \right. \right. \right. \right. \\ \left. \left. \left. \sqrt{2} a^{3/2} \operatorname{Cosh}[x] \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]} \right] \right) \right) \right) \\ \left. \left. \sqrt{a + b \operatorname{Sech}[x]^2} \right) \right) / \left(\sqrt{a + b} (a + 2b + a \operatorname{Cosh}[2x])^{3/2} \right)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^3}{\sqrt{a + b \operatorname{Sech}[x]^2}} dx$$

Optimal (type 3, 42 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}} \right]}{\sqrt{a}} + \frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{b}$$

Result (type 3, 105 leaves):

$$\left(\sqrt{a + 2b + a \operatorname{Cosh}[2x]} \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]} \right] \operatorname{Sech}[x] \right) / \\ \left(\sqrt{2} \sqrt{a} \sqrt{a + b \operatorname{Sech}[x]^2} \right) + \frac{(a + 2b + a \operatorname{Cosh}[2x]) \operatorname{Sech}[x]^2}{2b \sqrt{a + b \operatorname{Sech}[x]^2}}$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]}{\sqrt{a + b \operatorname{Sech}[x]^2}} dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \operatorname{Sech}[x]^2}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 70 leaves):

$$\left(\sqrt{a + 2b + a \operatorname{Cosh}[2x]} \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a + 2b + a \operatorname{Cosh}[2x]} \right] \operatorname{Sech}[x] \right) / \\ \left(\sqrt{2} \sqrt{a} \sqrt{a + b \operatorname{Sech}[x]^2} \right)$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \operatorname{Sech}[x]^2}} dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Tanh}[x]}{\sqrt{a+b \operatorname{Tanh}[x]^2}}\right]}{\sqrt{a}}$$

Result (type 3, 69 leaves):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \operatorname{Sinh}[x]}{\sqrt{a+2 b+a \operatorname{Cosh}[2 x]}}\right] \sqrt{a+2 b+a \operatorname{Cosh}[2 x]} \operatorname{Sech}[x]}{\sqrt{2} \sqrt{a} \sqrt{a+b \operatorname{Sech}[x]^2}}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]}{\sqrt{a + b \operatorname{Sech}[x]^2}} dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 124 leaves):

$$\left(\sqrt{a+2 b+a \operatorname{Cosh}[2 x]} \left(-\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a+b} \operatorname{Cosh}[x]}{\sqrt{a+2 b+a \operatorname{Cosh}[2 x]}}\right] + \sqrt{a+b} \operatorname{Log}\left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[x] + \sqrt{a+2 b+a \operatorname{Cosh}[2 x]}\right] \right) \operatorname{Sech}[x] \right) / \left(\sqrt{2} \sqrt{a} \sqrt{a+b} \sqrt{a+b \operatorname{Sech}[x]^2} \right)$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^3}{(a + b \operatorname{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{a+b}{a b \sqrt{a+b \operatorname{Sech}[x]^2}}$$

Result (type 3, 103 leaves):

$$\left(\left(-\frac{2\sqrt{a}(a+b)\operatorname{Cosh}[x](a+2b+a\operatorname{Cosh}[2x])}{b} + \sqrt{2}(a+2b+a\operatorname{Cosh}[2x])^{3/2} \right. \right. \\ \left. \left. \operatorname{Log}\left[\sqrt{2}\sqrt{a}\operatorname{Cosh}[x] + \sqrt{a+2b+a\operatorname{Cosh}[2x]}\right] \right) \operatorname{Sech}[x]^3 \right) / \left(4a^{3/2}(a+b\operatorname{Sech}[x]^2)^{3/2} \right)$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]^2}{(a+b\operatorname{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a}\operatorname{Tanh}[x]}{\sqrt{a+b\operatorname{Tanh}[x]^2}}\right]}{a^{3/2}} - \frac{\operatorname{Tanh}[x]}{a\sqrt{a+b\operatorname{Tanh}[x]^2}}$$

Result (type 3, 105 leaves):

$$- \left(\left(\operatorname{Sech}[x]^3 \left(-\sqrt{2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\operatorname{Sinh}[x]}{\sqrt{a+2b+a\operatorname{Cosh}[2x]}}\right] (a+2b+a\operatorname{Cosh}[2x])^{3/2} + \right. \right. \right. \\ \left. \left. \left. a^{3/2}\operatorname{Sinh}[x] + 4\sqrt{a}b\operatorname{Sinh}[x] + a^{3/2}\operatorname{Sinh}[3x] \right) \right) \right) / \left(4a^{3/2}(a+b\operatorname{Sech}[x]^2)^{3/2} \right)$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tanh}[x]}{(a+b\operatorname{Sech}[x]^2)^{3/2}} dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b\operatorname{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{1}{a\sqrt{a+b\operatorname{Sech}[x]^2}}$$

Result (type 3, 98 leaves):

$$- \left(\left((a+2b+a\operatorname{Cosh}[2x]) \right. \right. \\ \left. \left(2\sqrt{a}\operatorname{Cosh}[x] - \sqrt{2}\sqrt{a+2b+a\operatorname{Cosh}[2x]}\operatorname{Log}\left[\sqrt{2}\sqrt{a}\operatorname{Cosh}[x] + \sqrt{a+2b+a\operatorname{Cosh}[2x]}\right] \right) \right. \\ \left. \left. \operatorname{Sech}[x]^3 \right) \right) / \left(4a^{3/2}(a+b\operatorname{Sech}[x]^2)^{3/2} \right)$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Coth}[x]}{(a+b\operatorname{Sech}[x]^2)^{5/2}} dx$$

Optimal (type 3, 109 leaves, 9 steps):

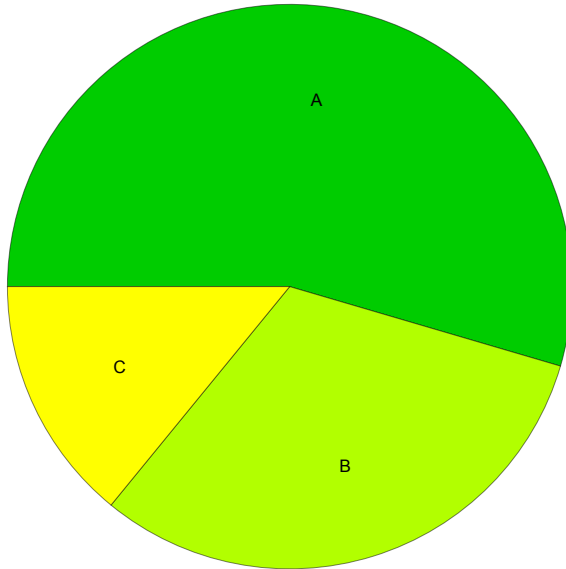
$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a}}\right]}{a^{5/2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sech}[x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{b}{3 a (a+b) (a+b \text{Sech}[x]^2)^{3/2}} - \frac{b (2 a+b)}{a^2 (a+b)^2 \sqrt{a+b \text{Sech}[x]^2}}$$

Result (type 3, 242 leaves):

$$\left(\left(-\frac{1}{3 a^2 (a+b)^2} 2 b \text{Cosh}[x] (a+2 b+a \text{Cosh}[2 x]) (7 a^2+16 a b+6 b^2+a (7 a+4 b) \text{Cosh}[2 x]) - \left((a+2 b+a \text{Cosh}[2 x])^{5/2} \left(\sqrt{a} (a^2-2 a b-b^2) \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a+b} \text{Cosh}[x]}{\sqrt{a+2 b+a \text{Cosh}[2 x]}}\right] + (a+b)^2 \left(\sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{2 a+2 b} \text{Cosh}[x]}{\sqrt{a+2 b+a \text{Cosh}[2 x]}}\right] - 2 \sqrt{a+b} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cosh}[x] + \sqrt{a+2 b+a \text{Cosh}[2 x]}\right] \right) \right) \right) \right) / \left(\sqrt{2} a^{5/2} (a+b)^{5/2} \right) \text{Sech}[x]^5 \Big/ \left(8 (a+b \text{Sech}[x]^2)^{5/2} \right)$$

Summary of Integration Test Results

220 integration problems



A - 120 optimal antiderivatives

B - 69 more than twice size of optimal antiderivatives

C - 31 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts